Estimating Candidate Valence*

Kei Kawai†
U.C. Berkeley

Takeaki Sunada‡
University of Rochester

October 29, 2021

Abstract

We estimate valence measures for candidates running in U.S. House elections from data on vote shares. Our estimates control for endogeneity of campaign spending and sample selection of candidates due to endogenous entry. Our identification and estimation strategy builds on ideas developed for estimating production functions. We find that incumbents have substantially higher valence measures than challengers running against them, resulting in about 9.4 percentage-point differences in the vote share, on average. We find that many of the best open-seat challengers are comparable to incumbents in terms of valence measures. We find that the differences in candidate valence between incumbents and challengers account for about 70% of the incumbency advantage, while differences in campaign spending account for about 30%.

Key words: Candidate Valence, Production Function, Incumbency Advantage
JEL classification: D72, D24

*We thank Daniel Diermeier, Igal Hendel, Aviv Nevo, Walter Stone, and Michael Whinston, for insightful comments.

†Department of Economics, U.C. Berkeley Email: kei@berkeley.edu
‡Simon Business School, University of Rochester Email: tsunada@simon.rochester.edu

1 Introduction

In many models of political economy, political candidates are horizontally differentiated through ideology and vertically differentiated through valence. While the development of the DW-NOMINATE scores for members of the U.S. Congress (Poole and Rosenthal 1985) has made possible testing theories about political ideology, a corresponding metric for candidate valence is still lacking. Although the vote share of each candidate should, in principle, be informative about candidate valence, endogeneity of campaign activities and sample selection arising from endogenous entry and exit make identification of candidate valence difficult. In this study, we build on empirical strategies used to estimate production functions to identify and estimate a metric of candidate valence from data on electoral outcomes. Our measure of valence is in units of vote share, capturing the differences in expected vote shares across candidates holding everything else constant. We use our approach to estimate the valence of each candidate running for U.S. House elections between 1984 and 2002.

We model vote shares in each election as a random variable that is determined, in part, by candidates' endogenous campaign spending and their valence. The valence of the candidate corresponds to a candidate specific constant (unobservable to the researcher) in the equation that determines votes. Our identification and estimation of candidate valence builds on ideas used in the literature on production functions because there is a natural parallel between recovering candidate valence from vote shares and recovering unobserved firm-level productivity from firm output. Specifically, we control for both the endogeneity of campaign spending and the selection of candidates entering each election using an approach developed in Olley and Pakes (1996). We embed the model of vote shares into a dynamic game with spending, fund-raising, savings, challenger entry and incumbent retirement and show that the candidates' policy functions are one-to-one between the valence and the observed actions. The one-to-one property allows us to recover the valence of incumbents as a function of their actions. We also use the model of challengers' entry to derive a sufficient statistic for the distribution of valence of challengers that choose to enter. The structure of the dynamic game allows us to link the vote shares to the valence of the candidates while controlling for endogeneity of spending and selection of candidates that choose to compete.

Our estimates suggest that there are substantial differences in the valence measures between incumbents and challengers. We find that the average valence measures of incumbents is about 9.4 percentage points higher in terms of vote share than those of challengers who run against incumbents. We also find larger dispersion of valence measures among challengers than among incumbents. The inter-quartile range of valence measures among incumbents is about 2.9 percentage points, whereas that among challengers is about 9.9 percentage points. Our findings are consistent with the fact that incumbents are selected partly by valence.

Regarding open-seat candidates, we find that the upper tail of the valence distribution resembles that of the incumbents. However, unlike the valence distribution of the incumbents, there is a substantial fraction of low valence candidates that increases the dispersion of the distribution. The mean valence measure of open-seat candidates is about 5.6 points higher than that of challengers that run against incumbents. The inter-quartile range is about 6.4 percentage points.

In order to illustrate a potential use of our metric, we study incumbency advantage in U.S. House elections. In particular, using the valence measures of candidates as outcome variables, we build on the regression discontinuity design used in Lee (2008) to identify the incumbency effect that can be attributed to differences in candidate valence. We also use the regression discontinuity design to identify the incumbency effect that is attributable to differential spending between incumbents and challengers. We hence offer a decomposition of the incumbency advantage identified in Lee (2008). Our results imply that about 67 percent of the incumbency advantage is explained by differences in valence and the remaining 33 percent is explained by differences in spending. Our result that spending accounts for a small portion of the incumbency advantage indicates that policy interventions designed to reduce incumbency advantage through the spending channel, such as subsidizing challengers' campaigns, may have limited effectiveness.

Literature Candidate valence plays an important role in many models of political competition. Differences in the valence of candidates affect convergence of platforms between candidates and alignment of policy with the preferences of the voters (Aragones and Palfrey 2004, Carter and Patty 2015, Buisseret and Weelden 2021). Candidate valence also plays a significant role in models of political selection (e.g., Snyder and Ting 2011, Serra 2011, Adams and Merrill III 2008). Despite their importance, measures of candidate valence that go beyond an index of observable candidate characteristics have been mostly lacking. Existing valence measures use characteristics such as candidates' occupation, political experience, legislative accomplishment, etc. (e.g., Green and Krasno 1988, Maestas and Rugeley 2008), but these measures miss potentially important unobservable components of valence. Other existing measures of valence are based on surveys (e.g., Stone et. al. 2010, and Stone and Simas 2010), but survey-based measures typically do not allow for a straightforward interpretation of their magnitude. Because our measures of valence correspond to candidate fixed effects in the model of vote shares, they capture any observable and unobservable candidate specific characteristics that affect votes. In addition, our measures are defined in units of vote share, allowing for a straightforward interpretation of their magnitude.

An important part of our empirical exercise is to separately identify the effect of campaign spending, candidate valence and other district characteristics on the vote share. In this regard, our paper is related to the extensive literature that estimates the causal effect of candidate spending on the vote share, including Jacobson (1978), Green and Krasno (1988), Levitt (1994), Gerber (1998), Erikson and Palfrey (2000), and da Silveira and de Mello (2011).[1] The key difference between our paper and previous work is that we focus on identifying candidate valence. Much of the previous work has treated candidate valence as nuisance parameters, for example, by differencing them out.

Our paper estimates a dynamic game between incumbents and challengers. Estimation of dynamic games in political economy include Merlo (1997), Diermeier et. al. (2003), Lim and Yurukoglu (2018), Sieg and Yoon (2017), Canen et. al. (2018), and Garcia-Jimeno and Yildirim (2018). Merlo (1997) and Diermeier et. al. (2003) study dynamic bargaining between parties during the formulation of a new government. Lim and Yurukoglu (2018) analyze repeated interactions between regulators and utility suppliers where regulator's political ideology influences the outcome. Sieg and Yoon (2017) estimate the impact of term limits in U.S. gubernatorial elections. Canen et. al. (2018) study political polarization and the role of party discipline in the U.S. Congress. Garcia-Jimeno and Yildirim (2018) analyze how media coverage interacts with candidates' ideological positioning.[2]

Finally, our paper contributes to the study of incumbency advantage. Starting from the early work of Erikson (1971), various approaches have been used to identify the incumbency advantage in U.S. Congressional elections. Gelman and King (1990) and Levitt and Wolfram (1997) use panel data methods. Ansolabehere et. al. (2000) use legislative re-

[1] See Stratmann (2005) for a survey.

[2] Other related work includes Iaryczower et. al. (2018) who study how candidates' preferences for office and ideology affect electoral accountability, Diermeier et. al. (2005), who study the value of a congressional seat and how congressional wage and term limits affect career decisions, and Gowrisanakaran et. al. (2008), who study how term limits influence voters' decisions.

districting and Lee (2008) uses regression discontinuity around a vote share of 50%.[3] Our analysis of incumbency advantage in Section 6 is based on Lee (2008).

2 Model

Overview We embed a model of vote shares in a dynamic model of U.S. House elections with endogenous spending, saving, entry and retirement decisions. In each period t ($t = 1, 2, ...\infty$), there is a stage game which is either an election with an incumbent seeking re-election or an open-seat election. In an election with an incumbent, potential challengers from the out-party (i.e., not the incumbent's party) decide whether or not to enter, and conditional on challenger entry, the incumbent and the challenger simultaneously make spending, saving and fund-raising decisions. We model the vote share as a function of the spending and the valence of the candidates, state variables (such as district characteristics) and a random shock. The winner becomes the incumbent next period. An open-seat election is the same as an election with an incumbent except that challengers from both parties make entry decisions. The time between the periods is two years, because Congressional elections take place every two years.

Sequence of Events within the Stage Game In an election with an incumbent, events occur in the following order:

1. Nature draws $N \in \{0, 1, 2, \cdots\}$, the number of potential challengers from the out-party according to a distribution F_N. The valence (i.e. quality) of the potential candidates, $\{q_{C,1}, q_{C,2}, ..., q_{C,N}\}$, are drawn independently according to F_{q_C}. We do not consider entry for the incumbent's party.[4] Each potential challenger observes the current state, the valence of the incumbent, and her own valence. Potential challengers simultaneously make entry decisions by comparing the value of entering and that of staying out. Upon entry, a challenger pays an entry cost κ.

2(a). If M ($1 \leq M \leq N$) potential challengers enter, an entrant with valence $q_{C,m}$ is selected to be the party nominee with probability $\pi(q_{C,m}, \mathbf{q}_{C,-m})$, where $\mathbf{q}_{C,-m} =$

[3] Levitt and Wolfram (1997) decompose the incumbency advantage into direct officeholder benefits, the ability of incumbents to scare-off high quality challengers, and higher average quality of incumbents vis-à-vis the typical open-seat candidate. Ansolabehere et. al. (2000) separates the electoral benefits of "homestyle" from other sources of incumbency advantage.

[4] Almost all incumbents in our sample become the party nominee, barring a major scandal. See Online Appendix 8.5 for the set of elections we drop due to scandals.

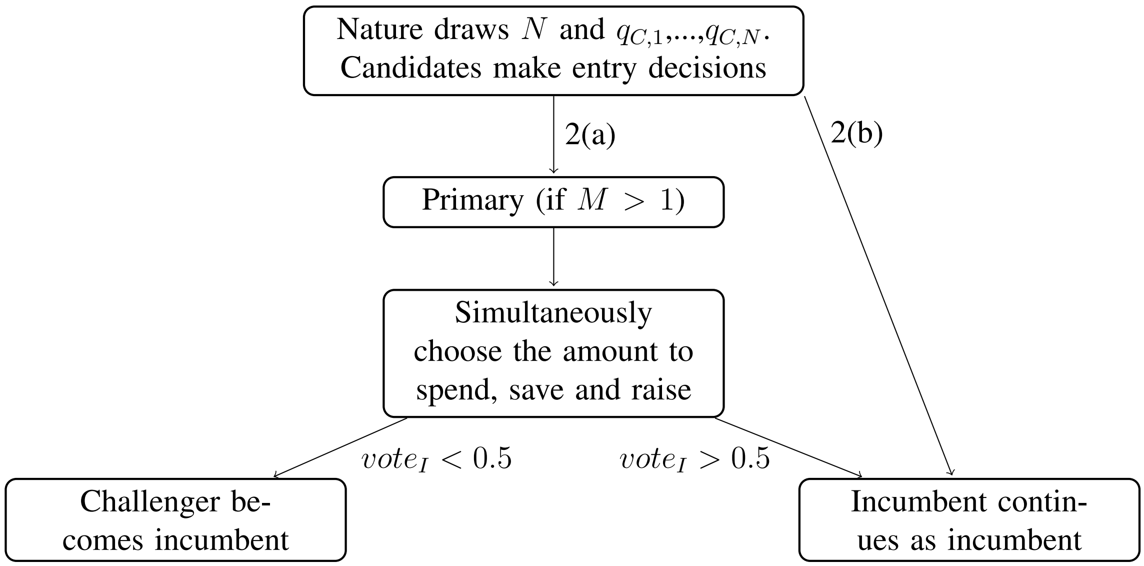

Figure 1: Timeline of the Stage Game for an Elections with an Incumbent

$(q_{C,1}, \ldots, q_{C,m-1}, q_{C,m+1}, \ldots, q_{C,M})$. $\pi(\cdot)$ represents the probability of winning the Primary. If exactly one challenger enters, that challenger becomes the party nominee with probability one.

In the general election, the incumbent and the nominee of the out-party simultaneously decide how much to spend, raise and save, taking as given own and opponent's valence. The vote shares are determined as a function of the spending and the valence of the candidates, state variables and a random shock.

2(b). If no potential challenger enters, the incumbent decides how much to spend, raise and save, and the incumbent becomes the winner with probability one.

3. The winner of the election receives utility B. State variables such as the incumbent's war chest and district characteristics evolve from current values to the next. Before the start of the next period, the winner chooses to retire or run for reelection. Conditional on running for reelection, the winner becomes the incumbent next period with war chest determined by the amount of money she saved in the previous period. If the incumbent retires, the stage game of the next period becomes an open-seat election.

Figure 1 illustrates the timeline of an election with an incumbent. $vote_I$ denotes the incumbent's vote share.

In open-seat elections, potential challengers from both parties make simultaneous entry decisions and the candidate selection process described in steps 1 and 2 applies to both parties. Once a single candidate is selected from each party, they compete in the general election in a way analogous to elections with incumbents. Because the model of open-seat elections is similar to that of elections with incumbents, we focus on the case of elections with incumbents in the following discussion. A full description of our model of open-seat elections is given in Online Appendix 8.1.

The Vote Share Equation We begin by describing how vote shares are determined in the general election (last step of case 2(a)). We specify the vote share equation as follows:

$$vote_I = \beta_I \ln d_I + \beta_C \ln d_C + \beta_{ten} \ln ten_I + \beta_X X + q_I - q_C + \varepsilon, \tag{1}$$

where $vote_I$ is the incumbent's vote share, $\ln d_I$ is the log spending (disbursement) of the incumbent and $\ln d_C$ is the log spending of the challenger. $\ln ten_I$ is the log tenure of the incumbent (i.e., the number of consecutive terms in office), X is a vector of exogenous control variables such as district characteristics, q_I and q_C are the valence of the incumbent and the challenger, and ε is a random shock.

The valence terms, q_I and q_C, capture the candidate's ability to attract votes. They may include candidates' personal traits such as name recognition, perceived leadership skills, public speaking skills, etc. They are unobserved to the researcher but observed by the candidates. Because candidates observe q_I and q_C when making their decisions, $\ln d_I$ and $\ln d_C$ are potentially correlated with q_I and q_C. On the other hand, ε is not observed to the candidates and hence orthogonal to their decisions.

We assume that the error term, ε, follows a Normal distribution with mean 0.5 and variance σ_ε^2. The probability that the incumbent wins, i.e. the probability that vote share of the incumbent exceeds 0.5, can be written as follows:

$$\Pr(vote_I > 0.5) = \Phi\left(\frac{1}{\sigma_\varepsilon}(\beta_I \ln d_I + \beta_C \ln d_C + \beta_{ten} \ln ten_I + \beta_X X + q_I - q_C)\right), \tag{2}$$

where Φ is the c.d.f. of a standard Normal distribution.

Incumbent's Problem Consider the problem of the incumbent when the incumbent faces a challenger (after the Primary in case 2(a)). For a given strategy of the challenger, a contested incumbent solves the following dynamic programming problem:

$$\begin{aligned} v_I(\mathbf{s}, q_C) = & \max_{d_I \geq 0, w_I' \geq 0} u_I + \delta \Pr(vote_I(\mathbf{s}, q_C) > 0.5) \mathbb{E}_{X'|X}[V_I(\mathbf{s}')], \\ \text{where } u_I = & B \cdot \Pr(vote_I(\mathbf{s}, q_C) > 0.5) - C_I(w_I' + d_I - w_I; q_I) + H_I(d_I). \end{aligned} \tag{3}$$

The incumbent chooses the amount of spending, d_I, and the amount of savings, w_I', given the challenger's valence q_C and the state $\mathbf{s}$, where $\mathbf{s} = \{q_I, w_I, ten_I, X\}$. The term w_I is the war chest of the incumbent at the beginning of the period. The flow payoff, u_I, consists of three terms. B is the utility from winning and it is multiplied by the probability of winning. The term $C_I(\cdot)$ captures the costs that the incumbent incurs from raising money. The amount raised by the incumbent is the sum of future savings, w_I', and spending, d_I, less the war chest, w_I. We let $C_I(\cdot)$ depend on q_I and assume that the marginal cost of raising money is strictly decreasing in q_I, so that candidates with higher valence have lower marginal cost of raising money. $H_I(\cdot)$ represents the consumption value of spending. It captures the fact that campaign spending can sometimes benefit candidates directly, for example, through purchases of personal articles.[5]

The second term of the value function v_I is the continuation value which is a product of the discount factor δ, the probability of winning, and the next period's (ex-ante) value function $\mathbb{E}_{X'|X}[V_I(\mathbf{s}')]$ we define below. The expectation of the next period's value function is taken with respect to X', the realization of X next period.

We assume that X follows an exogenous Markov process. We assume a deterministic transition for q_I, w_I and ten_I. The incumbent war chest in the next period equals the amount that the incumbent saves in the current period plus 10% interest.[6] The tenure of the incumbent increases by 1 as $ten_I' = ten_I + 1$. Finally, we assume that q_I is constant over time. While this is restrictive, we can account for deterministic trends in electoral strength through ten_I. Allowing for q_I to evolve stochastically is conceptually straightforward, but the estimation of such a model becomes data-intensive. We discuss this point in detail in Online Appendix 8.7.

When no challenger enters (case 2(b)), the election is uncontested and the incumbent wins with probability one. The problem of the incumbent in an uncontested election is as

[5] $H_I(\cdot)$ accounts for the fact that there is incumbent spending even in periods when the incumbent seems almost certain to win. In the estimation, we allow for the possibility that $H_I(\cdot) = 0$, however.

[6] Since the time between periods is two years, a 10% interest implies an annual interest of about 5%.

follows:

$$\tilde{v}_I(\mathbf{s}) = \max_{d_I \geq 0, w_I' \geq 0} \tilde{u}_I + \delta \mathbb{E}_{X'|X}[V_I(\mathbf{s}')],$$
$$\text{where } \tilde{u}_I = B - \tilde{C}_I(w_I' + d_I - w_I; q_I) + \tilde{H}_I(d_I). \tag{4}$$

The term $\tilde{u}_I$ is the period utility of the incumbent when she is uncontested, and the expression is obtained by replacing $\Pr(vote_I(\mathbf{s}, q_C) > 0.5)$ with 1, and by replacing $C_I(\cdot)$ and $H_I(\cdot)$ with $\tilde{C}_I(\cdot)$ and $\tilde{H}_I(\cdot)$ in expression (3). $\tilde{C}_I(\cdot)$ and $\tilde{H}_I(\cdot)$ are the costs of raising money and the consumption value from spending in uncontested periods, respectively. We assume that the marignal cost of raising money is strictly decreasing in q_I.

The incumbent's ex-ante value function at the beginning of the stage game is as follows:

$$V_I(\mathbf{s}) = (1 - \lambda(\mathbf{s}))(1 - P_e(\mathbf{s}))\tilde{v}_I(\mathbf{s}) + (1 - \lambda(\mathbf{s}))P_e(\mathbf{s}) \int_{qc} v_I(\mathbf{s}, q_C) dG_{q_C}(q_C|\mathbf{s}), \tag{5}$$

where $P_e(\mathbf{s})$ is the probability that a challenger enters, and $\lambda(\mathbf{s})$ is the probability that the incumbent retires. The value function consists of two terms: the first is the value of the incumbent when she is uncontested and the second is the value when she is contested. We assume that if an incumbent retires, she receives zero payoff thereafter. Because the challenger's valence, q_C, is uncertain at the beginning of the period, we take the expectation of $v_I(\mathbf{s}, q_C)$ with respect to q_C. We denote its distribution by $G_{q_C}(\cdot|\mathbf{s})$.

Note that $P_e(\mathbf{s})$ and $G_{q_C}(\cdot|\mathbf{s})$ are equilibrium objects that are endogenously determined by the challengers' entry decisions. We describe how $P_e(\mathbf{s})$ and $G_{q_C}(\cdot|\mathbf{s})$ are determined by the problem of the challenger below. Although we do not explicitly model incumbents' retirement decisions, $\lambda(\mathbf{s})$ can be interpreted as the policy function of the incumbent with respect to retirement that results from the incumbent's optimizing behavior (See e.g., Diermeier et. al. (2005) for a model of retirement).

Challenger's Problem The problem of a challenger who competes in the general election is given as follows:

$$v_C(\mathbf{s}, q_C) = \max_{d_C \geq 0, w_C' \geq 0} B \cdot \Pr(vote_I(\mathbf{s}, q_C) < 0.5) - C_C(w_C' + d_C, q_C)$$
$$+ H_C(d_C) + \delta \Pr(vote_I(\mathbf{s}, q_C) < 0.5) \mathbb{E}_{X'|X}[V_I(\mathbf{s}')]. \tag{6}$$

The challenger chooses the amount of spending, d_C, and the amount of savings, w'_C, given her own valence q_C and s, where $\mathbf{s} = \{q_I,\, w_I,\, ten_I,\, X\}$. $C_C(\cdot)$ and $H_C(\cdot)$ capture the challenger's cost of fund-raising and personal benefit from spending, respectively. The challenger's next-period value function is the same as that of the incumbent, V_I, because the challenger becomes the incumbent if she wins. The next-period value, V_I, depends on the valence of the challenger, savings from period t (plus 10% interest), tenure $(= 2)$ and the vector of exogenous variables, X', so that $\mathbf{s}' = \{q_C,\, 1.1w'_C,\, 2,\, X'\}$.[7]

We now consider the challenger's entry decision. We assume that each challenger makes an entry decision by comparing the value of entering with the cost of entry, κ. The value of entering is the product of the value function of a challenger in the general election, $v_C(\mathbf{s}, q_C)$, and the probability of winning the primary, which we denote by $p(\mathbf{s}, q_C)$. The probability of winning the primary can be expressed as follows:

$$p(\mathbf{s}, q_{C,m}) = \mathbb{E}_M \left[\int \pi(q_{C,m}, \mathbf{q}_{C,-m}) dF_{\mathbf{q}_{C,-m}}(\mathbf{q}_{C,-m}|M, \mathbf{s}) \middle| \mathbf{s} \right], \tag{7}$$

where $\pi(q_{C,m}, \mathbf{q}_{C,-m})$ is the probability that entrant m wins the primary if her valence measure is $q_{C,m}$ and her opponents' valence measures are $\mathbf{q}_{C,-m}$. Because we do not explicitly model the Primary, we let $\pi(\cdot)$ be flexible, only requiring $\pi(\cdot)$ to be symmetric in $\mathbf{q}_{C,-m}$ and increasing in own valence.[8] The challenger does not know the realization of the number of total entrants, M, or the valence measures of her opponents, $\mathbf{q}_{C,-m}$, when making her entry decision. We hence integrate $\pi(q_{C,m}, \mathbf{q}_{C,-m})$ with respect to $\mathbf{q}_{C,-m}$ and M, to obtain the expression for $p(\mathbf{s}, q_C)$. We denote by $F_{\mathbf{q}_{C,-m}}(\mathbf{q}_{C,-m}|M, \mathbf{s})$ the distribution of the opponents' valence measures conditional on M and the state s. The distribution of $\mathbf{q}_{C,-m}$ and M are both endogenous.

Each potential challenger chooses to enter if the value of entry, $p(\mathbf{s}, q_C)v_C(\mathbf{s}, q_C)$, is higher than the entry cost, κ. This implies that, if $p(\mathbf{s}, q_C)v_C(\mathbf{s}, q_C)$ is increasing in q_C, the entry decision of a challenger can be expressed by the following cutoff rule:[9]

[7]Because we define vote share equation as a function of log tenure, tenure of each candidate starts from 1 in her first election.

[8]When exactly one potential challenger enters $(M = 1)$, the entrant becomes the party nominee with probability 1.

[9]Because we assume that $\pi(q_{C,m}, \mathbf{q}_{C,-m})$ is increasing in $q_{C,m}$, $p(\mathbf{s}, q_C)$ is increasing in q_C by assumption. Hence $p(\mathbf{s}, q_C)v_C(\mathbf{s}, q_C)$ is increasing in q_C when $v_C(\mathbf{s}, q_C)$ is not too decreasing in q_C. In the Online Appendix 8.8, we show that at the estimated parameter values, $v_C(\mathbf{s}, q_C)$ is increasing in q_C for about 90% of the challengers in our data.

$$\chi(\mathbf{s}, q_C) = \begin{cases} 1\text{: if } q_C > \bar{q}_C(\mathbf{s}) \\ [0,1]\text{: if } q_C = \bar{q}_C(\mathbf{s}) \\ 0\text{: if } q_C < \bar{q}_C(\mathbf{s}) \end{cases},$$

where $\bar{q}_C(\mathbf{s})$ is defined implicitly as the solution to $p(\mathbf{s}, \cdot)v_C(\mathbf{s}, \cdot) - \kappa = 0$. $\bar{q}_C$ is the type of challenger that is indifferent between entering and not entering.

Equilibrium In order to close the model, we express the equilibrium entry probability, $P_e(\mathbf{s})$ and the equilibrium valence distribution of the challengers, $G_{q_C}(\cdot|\mathbf{s})$, as functions of the challengers' entry threshold, $\bar{q}_C(\mathbf{s})$. We then define the equilibrium of the game as a fixed point in the space of the challengers' entry threshold, $\bar{q}_C(\mathbf{s})$.

We start with the equilibrium entry probability, $P_e(\mathbf{s})$. Given that the entry probability is equal to one minus the probability of no entry, it can be expressed using $\bar{q}_C(\mathbf{s})$ as follows:

$$P_e(\mathbf{s}) = \mathbb{E}_N \left[1 - F_{q_C}(\bar{q}_C(\mathbf{s})^N \middle| \mathbf{s} \right], \tag{8}$$

where the expectation is taken with respect to the distribution of N.

The expression for the equilibrium valence distribution of the challengers, $G_{q_C}(\cdot|\mathbf{s})$ is as follows:

$$\begin{aligned} G_{q_C}(t|\mathbf{s}) &= \mathbb{E}_N \left[\mathbb{E}_M \left[\Pr(\text{Valence of Primary winner} \leq t | M, \mathbf{s}) | N, \mathbf{s} \right] \middle| \mathbf{s} \right] \quad (9) \\ &= \mathbb{E}_N \left[\mathbb{E}_M \left[M \int_{\bar{q}_C}^{t} \int \pi(q_C, \mathbf{q}_{C,-m}) dF_{\mathbf{q}_{C,-m}}(\mathbf{q}_{C,-m}|M, \mathbf{s}) \frac{dF_{q_C}}{1 - F_{q_C}(\bar{q}_C)} \middle| N, \mathbf{s} \right] \middle| \mathbf{s} \right] \\ &= \mathbb{E}_N \left[\mathbb{E}_M \left[M \frac{\int_{\bar{q}_C}^{t} \int_{\bar{q}_C}^{+\infty} \cdots \int_{\bar{q}_C}^{+\infty} \pi(q_C, \mathbf{q}_{C,-m})(dF_{q_C})^M}{(1 - F_{q_C}(\bar{q}_C))^M} \middle| N, \mathbf{s} \right] \middle| \mathbf{s} \right] \\ &= \mathbb{E}_N \left[\sum_{M=1}^{N} Bin(N, M; 1 - F_{q_C}(\bar{q}_C)) M \frac{\int_{\bar{q}_C}^{t} \int_{\bar{q}_C}^{+\infty} \cdots \int_{\bar{q}_C}^{+\infty} \pi(q_{C,n}, \mathbf{q}_{C,-n})(dF_{q_C})^M}{(1 - F_{q_C}(\bar{q}_C))^M} \middle| \mathbf{s} \right], \end{aligned}$$

where we suppress the dependence of $\bar{q}_C$ on s. To go from the first to the second line, we use the fact that when M challengers enter, each candidate with valence q_C wins the primary with probability $\int \pi(q_{C,m}, \mathbf{q}_{C,-m}) dF_{\mathbf{q}_{C,-m}}(\mathbf{q}_{C,-m}|M, \mathbf{s})$. Hence, the probability that a given entrant that wins the Primary has valence less than t is the integral of this expression from $\bar{q}_C$ to t. Because the event that each one of M entrants wins the Primary is disjoint, we multiply this expression by M to obtain the probability that the Primary winner has valence less than t.[10] To derive the third line, we use the assumption that q_C are independently drawn from F_{q_C}.[11] To derive the fourth line, we use the fact that $M = \sum_{i=1}^{N} \chi(\mathbf{s}, q_{C,i})$: the number of challengers equals to the number of potential challengers whose $q_{C,i}$ exceeds $\bar{q}_C(\mathbf{s})$. The probability of observing M entrants given N potential challengers can then be expressed as $Bin(N, M; 1 - F_{q_C}(\bar{q}_C))$, where $Bin(n_1, n_2; p)$ is the probability that we have n_2 successes out of n_1 trials with success rate p.

We now define the equilibrium of the game. Formally, the players of the game are the incumbent and an infinite sequence of potential challengers. The strategies of the game are how much to spend, save, and raise for both the incumbent and the general election challenger, as well as the entry decisions of the potential challengers. The solution concept we use is stationary Markov Perfect Equilibria (Maskin and Tirole 1988).

We can think of the equilibrium of the game as a fixed point in the potential challenger's entry threshold, $\bar{q}_C(\cdot)$ $(\mathbf{s} \mapsto \mathbb{R})$. If we fix a $\bar{q}_C(\cdot)$, it uniquely determines $G_{q_C}(t|\mathbf{s})$ and $P_e(\mathbf{s})$. $G_{q_C}(t|\mathbf{s})$ is determined from expression (9) and $P_e(\mathbf{s})$ can be expressed as $P_e(\mathbf{s}) = \mathbb{E}_N \left[1 - F_{q_C}(\bar{q}_C)^N \middle| \mathbf{s}\right]$. Once $G_{q_C}(t|\mathbf{s})$ and $P_e(\mathbf{s})$ are determined, expressions (3) through (6) define a dynamic game of spending, saving and fund-raising. Consider the value function of the challenger v_C associated with the solution of this dynamic game. The value function v_C defines a threshold for challenger entry given by $p(\mathbf{s}, \cdot)v_C(\mathbf{s}, \cdot) - \kappa = 0$, where $p(\mathbf{s}, \cdot)$ is the probability of winning the primary given in expression (7).[12] In equilibrium, the threshold for entry that solves this expression must coincide with the entry threshold that we fixed at the outset.

A Model of Open-seat Elections Although the model of open-seat elections is part of the dynamic game, a stage game is an open-seat election only when the current incumbent retires. Thus, open-seat elections do not appear in any of the continuation games for the incumbents and challengers in contested elections.

In an open-seat election, potential challengers from both parties make simultaneous entry decisions, and the candidate selection process for the out-party described above applies

[10] The probability that the valence of the Primary winner is less that t is the sum of the probabilities that candidate 1 wins and her valence is less than t, candidate 2 wins and her valence is less than t, and so on.

[11] When q_C are independently drawn from F_{q_C}, the valence distribution of the other challengers, $F_{\mathbf{q}_{C,-m}}(\mathbf{q}_{C,-m}|M, \mathbf{s})$, is obtained by a restriction of $F_{q_C} \times \cdots \times F_{q_C}$ to $[\bar{q}_C(\mathbf{s}), \infty]^{M-1}$

[12] By following steps similar to those for expression (9), we can express $p(\mathbf{s}, \cdot)$ in expression (7) as a function of $\bar{q}_C$ as follows:

$$p(\mathbf{s}, \cdot) = \mathbb{E}_N \left[\sum_{M=1}^{N} Bin(N, M; 1 - F_{q_C}(\bar{q}_C)) \frac{\int_{\bar{q}_C}^{+\infty} \cdots \int_{\bar{q}_C}^{+\infty} \pi(q_{C,m}, \mathbf{q}_{C,-m}) (dF_{q_C})^{M-1}}{(1 - F_{q_C}(\bar{q}_C))^{M-1}} \middle| \mathbf{s} \right].$$

to both parties. Once candidates are selected as the party nominee, the candidates compete in the general election in a way analogous to contested elections. One difference between the two is that the war chest of the two candidates is set to zero. We also permit open-seat specific marginal effect of spending, β_O, in the vote share equation. The winner of the open-seat election becomes the incumbent next period. The continuation value associated with winning an open-seat election is V_I. The Online Appendix 8.1 contains a full description of the model of open-seat elections.

Deriving Model Properties Used in Identification We now discuss two properties of the model that we exploit in identification. The first property is the injectivity of the policy function of uncontested incumbents:

Proposition 1 (Injectivity): *Assume that the marginal cost of raising money, $\frac{\partial}{\partial x}\tilde{C}_I(x, q_I)$, is strictly decreasing with respect to q_I. Then, the policy functions of an uncontested incumbent, $\{d_I(\mathrm{s}),w'_I(\mathrm{s})\}$, are one-to-one from q_I to (d_I, w'_I), holding other state variables fixed.*

Proof. See Online Appendix 8.3. ■

Proposition 1 states that, if we have $\mathrm{s} = \{q_I, w_I, ten_I, X\}$ and $\mathrm{s}' = \{q'_I, w_I, ten_I, X\}$ such that $q_I \neq q'_I$, but all of the other elements of the state s are the same, the actions assocaited with s and s′ must be different, i.e., $\{d_I(\mathrm{s}), w'_I(\mathrm{s})\} \neq \{d_I(\mathrm{s}'), w'_I(\mathrm{s}')\}$. The injectivity of the policy function allows us to invert the policy functions of uncontested incumbents and express q_I as a function of observed states and actions. This property is used to construct a control function for q_I when we consider identification of the vote share equation.

The second model property is the presence of sufficient statistics for the valence distribution of the challengers, $G_{q_C}(\cdot|\mathrm{s})$:

Proposition 2-1 (Sufficient statistic): *When the distribution of potential challengers, N, does not depend on* s*, the expected number of actual entrants in the primary,* $\Pr[M > 0|\mathrm{s}]$*, is a sufficient statistic for the valence distribution of the challengers, $G_{q_C}(\cdot|\mathrm{s})$.*

We say that $h = h(\mathrm{s})$ is a sufficient statistic for $f(\mathrm{s})$ if $h(\mathrm{s}') = h(\mathrm{s}'')$ implies $f(\mathrm{s}') = f(\mathrm{s}'')$. Proposition 2-1 states that $\Pr[M > 0|\mathrm{s}] = \Pr[M > 0|\mathrm{s}']$ implies $G_{q_C}(\cdot|\mathrm{s}) = G_{q_C}(\cdot|\mathrm{s}')$. Proposition 2-1 is a special case of the following proposition:

Proposition 2-2 (Sufficient statistic): *Suppose that N is distributed according to a CDF*

$F_N(\cdot|\mathbf{s})$ that is fully characterized by the first L moments, $\mathbb{E}[N|\mathbf{s}],\cdots,\ \mathbb{E}[N^L|\mathbf{s}]$. Generically, either (i) $\Pr[M>0|\mathbf{s}]$ and L moments of M, m_M=$\{\Pr[M>0|\mathbf{s}],\mathbb{E}[M|\mathbf{s}],\cdots,\ \mathbb{E}[M^L|\mathbf{s}]\}$ or (ii) $\Pr[M>0|\mathbf{s}]$ and $L+1$ moments of M, $\tilde{m}_M$=$\{\Pr[M>0|\mathbf{s}],\mathbb{E}[M|\mathbf{s}],\cdots,\ \mathbb{E}[M^{L+1}|\mathbf{s}]\}$ are sufficient statistics for $G_{q_C}(\cdot|\mathbf{s})$.

Note that many distributions are fully characterized by the first few moments. For example, binomial distributions are fully characterized by their means and second moments. Discrete uniform and discrete Normal distributions are also fully characterized by the first two moments. Poisson distributions are fully characterized by their mean. Proposition 2-2 states that, if the number of potential challengers, N, follows a distribution that is fully characterized by the first L moments, then $\Pr(M>0|\mathbf{s})$ and the first L or $L+1$ moments of M are sufficient statistics for $G_{q_C}(\cdot|\mathbf{s})$. Whether L moments or $L+1$ moments are required as sufficient statistics depends on whether or not the root of a particular system of polynomial equations is unique. If the root is unique, L moments suffice, but in general, $L+1$ moments are required. Appendix 8.2 specifies the system of polynomial equations that determine whether L or $L+1$ moments are needed.

Proof. We give a proof of Proposition 2-1 here. Appendix 8.2 proves Proposition 2-2. From expression (9), we can see that $\bar{q}_C(\mathbf{s})$ is a sufficient statistic for $G_{q_C}(\cdot|\mathbf{s})$. From expression (8), we can see that $\bar{q}_C(\mathbf{s})$ and $\Pr(M>0|\mathbf{s})$ are one-to-one. Hence, $\Pr(M>0|\mathbf{s})$ is also a sufficient statistic for $G_{q_C}(\cdot|\mathbf{s})$.

■

Propositions 2-1 and 2-2 allow us to control for the selection of challengers in identifying the vote share equation. In our empirical application, we use $\mathbb{E}[M|\mathbf{s}]$ and $P_e(\mathbf{s})$ as sufficient statistics for $G_{q_C}(\cdot|\mathbf{s})$.

3 Identification and Estimation

We aim to identify the realizations of q_I and q_C for each candidate, as well as the vote share equation and various components of the candidates' payoff functions, such as $C_I(\cdot)$ and $H_I(\cdot)$.[13] We first identify the vote share equation as well as the realization of q_I for a subset of the incumbents. We use the actions of the incumbents in past elections to construct a control function for q_I. We do not identify q_C in this step because we cannot construct

[13]We do not estimate some of the model primitives of the Primary, such as $\pi(\cdot)$, F_N, R, and κ. They are not necessary for recovering candidate valence measures, which is the focus of the paper.

an analogous control function for the challengers. We identify q_C and the candidates' payoffs by utilizing the first-order conditions associated with the candidates' problem. We also recover from the fist-order conditions the valence terms for the subset of incumbents whose valence terms were not identified from the vote share equation.

3.1 Identification of Incumbent's Valence and the Vote Share Equation

Recall that the vote share equation is specified as follows:

$$vote_I = \beta_I \ln d_I + \beta_C \ln d_C + \beta_{ten} \ln ten_I + \beta_X X + q_I - q_C + \varepsilon. \tag{1}$$

The two main challenges in identifying the vote share equation and candidate valence are sample selection bias and endogeneity of spending. Sample selection problem arises from the fact that the challengers' entry decisions and the incumbents' exit decisions are endogenous. Because the candidates know the state s when making entry and exit decisions, the valence measures of the candidates are potentially correlated with variables in s including those that evolve exogenously such as X and ten_I. Endogeneity of spending arises because the candidates choose d_I and d_C based, in part, on q_I and q_C. For example, incumbents typically spend more against a challenger with higher valence. Note that because our main goal is to identify the candidate valence measures, we cannot rely on panel data methods that sweep out the valence terms as nuisance parameters.

In order to overcome these challenges, we exploit the close analogy between our setting and estimation of production functions. In the context of production function estimation, the input decisions and entry/exit decisions of the firms depend on the unobserved (to the researcher) productivity of the firms. This dependence gives rise to endogeneity and sample selection bias similar to the one considered here. The control function approach developed by Olley and Pakes (1996) enables one to recover the production function and firm productivity measures even in the presence of endogeneity and sample selection problems. Because we have two unobservable terms (q_I and q_C) as opposed to just one in Olley and Pakes (1996), we adapt their approach to our setting. We use the control function to invert out q_I as a function of observable terms, whereas we fix the distribution of q_C using its sufficient statistics.

Control Function for q_I We use the inverse of the policy function of uncontested incumbents as a control function to express q_I as a function of observed variables. The policy functions associated with the problem of uncontested incumbents are how much to spend, $d_I(\mathbf{s})$, and how much to save, $w'_I(\mathbf{s})$. These policy functions can be viewed as mappings from q_I to (d_I, w'_I), holding the other state variables fixed. Because the mapping $q_I \mapsto (d_I, w'_I)$ is one-to-one (Proposition 1), we can uniquely solve for q_I using these policy functions as $q_I = q_I(\bar{\mathbf{s}}_U)$, where $\bar{\mathbf{s}}_U$ denotes the vector of state variables and actions in the uncontested period.

Because the functional form of the policy functions, $d_I(\mathbf{s})$ and $w'_I(\mathbf{s})$ depends on the primitives of the model, so does $q_I(\cdot)$. Hence, $q_I(\cdot)$ is not a known object. Nevertheless, the fact that we can express q_I as $q_I(\bar{\mathbf{s}}_U)$ allows us to substitute out q_I in the vote share equation with a nonparametric function of observables, $\bar{\mathbf{s}}_U$. This allows us to identify $q_I(\cdot)$ by tracing out how the vote share varies with $\bar{\mathbf{s}}_U$.[14][15] Once $q_I(\cdot)$ is identified, we can explicitly control for selection and endogeneity with respect to q_I.

Sample Selection Bias of q_C and Sufficient Statistics We next consider selection of challengers that choose to run against incumbents. Substituting out q_I with $q_I(\bar{\mathbf{s}}_U)$, the vote share equation can be expressed as follows.

$$\begin{aligned} vote_I = \beta_I \ln d_I + \beta_C \ln d_C + \beta_{ten} \ln ten_I + \beta_X X + q_I(\bar{\mathbf{s}}_U) \\ - \mathbb{E}[q_C|\mathbf{s}] - (q_C - \mathbb{E}[q_C|\mathbf{s}]) + \varepsilon, \end{aligned} \tag{1'}$$

where we have also decomposed q_C into a part that depends on $\mathbf{s}$ and a part that is orthogonal to $\mathbf{s}$. In our setup, endogenous entry creates a sample selection problem in that the state variables such as X and ten_I affect the vote share through their effect on challenger valence $\mathbb{E}[q_C|\mathbf{s}]$ in addition to the direct effect. Following Olley and Pakes (1996), we use sufficient statistics to identify the direct effect of ten_I and X on the vote share while holding fixed the sample selection effect.

As we showed in Proposition 2-2, m_M, the moments of the number of entrants in the Primary, are sufficient statistics for the distribution of the challenger's valence in the elec-

[14]Unlike in Olley and Pakes (1996) and Levinsohn and Petrin (2003), there are no collinearity issues when estimating the vote share equation (See Ackerberg Caves and Frazer 2006 and Gandhi, Navarro and Rivers 2020). This is because we express q_I as a function of actions and state variables in some period t which we then use to replace out q_I in the vote share equation of some future period $t' > t$.

[15]In our empirical application, if an incumbent experiences multiple uncontested elections, we use the observations from the first uncontested election.

tion, $G_{q_C}(\cdot|\mathbf{s})$. This implies that we can express $\mathbb{E}[q_C|\mathbf{s}]$ as a function of m_M as follows:

$$\begin{aligned}\mathbb{E}[q_C|\mathbf{s}] &= \mathbb{E}[q_C|m_M(\mathbf{s})] \\ &\equiv g(m_M(\mathbf{s})).\end{aligned} \tag{10}$$

Although $m_M(\mathbf{s})$ are endogenous objects, they are nonparametrically identified directly from the data. This is because the only component of s that is not directly observed is q_I; and q_I can be written as a function of observables as $q_I = q_I(\bar{\mathbf{s}}_U)$. In other words, we can express m_M as functions of $\bar{\mathbf{s}}_U \cup \mathbf{s} \backslash \{q_I\}$. Hence, the arguments of the function $g(\cdot, \cdot)$ in expression (10) are identified.

Replacing $\mathbb{E}[q_C|\mathbf{s}]$ as a function of m_M as in expression (10) allows us to control for the indirect effect of s due to selection. By exploiting variation in s that leaves m_M fixed, $\mathbb{E}[q_C|\mathbf{s}]$ remains constant and we can identify the direct effect of s on the vote share.

Endogeneity of Spending with Respect to q_C Lastly, we control for the endogeneity between $\{d_I, d_C\}$ and $(q_C - \mathbb{E}[q_C|\mathbf{s}])$. Because $(q_C - \mathbb{E}[q_C|\mathbf{s}])$ is the difference between the ex-post realization of the challenger's valence from its expectation, it is orthogonal to the set of predetermined variables, s. Hence we deal with the endogeneity by projecting the vote shares on s as follows:

$$\begin{aligned} vote_I =& \mathbb{E}[vote_I|\mathbf{s}] + \epsilon \\ =& \beta_I \mathbb{E}[\ln d_I|\mathbf{s}] + \beta_C \mathbb{E}[\ln d_C|\mathbf{s}] + \beta_{ten} \ln ten_I + \beta_X X \\ &+ q_I(\bar{\mathbf{s}}_U) - g(m_M) + \epsilon, \end{aligned} \tag{1''}$$

where $\epsilon \equiv (vote_I - \mathbb{E}[vote_I|\mathbf{s}])$. The term $\mathbb{E}[vote_I|\mathbf{s}]$ is the vote share equation evaluated *before* the challenger's valence q_C realizes. Hence $\mathbb{E}[\epsilon|\mathbf{s}] = 0$ by construction. In particular, ϵ is uncorrelated with $\mathbb{E}[\ln d_I|\mathbf{s}]$ and $\mathbb{E}[\ln d_C|\mathbf{s}]$. Because $\mathbb{E}[\ln d_I|\mathbf{s}]$ and $\mathbb{E}[\ln d_C|\mathbf{s}]$ are identified directly from the data, the orthogonality condition guarantees identification of β_I and β_C. Similarly, the orthogonality between ϵ and ten_I, X identifies β_{ten} and β_X. Variation in $\bar{\mathbf{s}}_U$ identifies $q_I(\cdot)$.[16]

The intuition behind the identification is as follows. Consider the case in which m_M consists of the expected number of Primary entrants, $\mathbb{E}[M|\mathbf{s}]$, and the probability that a

[16] To be precise, $q_I(\cdot)$ and $g(\cdot, \cdot)$ are identified up to an additive constant. In our environment, shifting up or down the valence measures of all of the candidates by the same amount does not change the distribution of observable outcomes. We normalize the sample average of $q_I(\cdot)$ to zero.

challenger enters to an election, $P_e(\mathbf{s})$. This is the specification we use in our empirical analysis.[17] Now, fix $\bar{\mathbf{s}}_U$, the vector of state variables and actions in an uncontested period. This is equivalent to fixing q_I. Now consider variation in s that keeps $\mathbb{E}[M|\mathbf{s}]$ and $P_e(\mathbf{s})$ constant. For example, let state $\mathbf{s} = \mathbf{s}_1$ be such that the incumbent starts with high war chest, but variables in X that affect vote share are not so favorable to the incumbent. Let state $\mathbf{s} = \mathbf{s}_2$ be such that the incumbent starts with low war chest, but variables in X are more favorable, so that $\mathbb{E}[M|\mathbf{s}]$ and $P_e(\mathbf{s})$ are the same. The sufficient statistic property guarantees that the mean challenger valence will be the same in $\mathbf{s}_1$ and $\mathbf{s}_2$. Hence it is possible to use the variation in expected candidate spending and X across $\mathbf{s}_1$ and $\mathbf{s}_2$ to identify the coefficients of the vote share equation. Once all of the coefficients are identified, variation in $\bar{\mathbf{s}}_U$ and $\mathbb{E}[M|\mathbf{s}]$ identifies $q_I(\cdot)$ and $g(\cdot)$.

Note that this approach requires that we observe the incumbents' actions in uncontested periods. Hence, in order to estimate the vote share equation, we only use a subset of elections in which the incumbent has experienced an uncontested election in the past.[18] We discuss identification of valence measures for incumbents who never experience uncontested elections in Section 3.2.

Extensions to Include Outside Spending and Lack of Uncontested Elections Our approach of estimating the vote share equation extends to settings with substantial outside spending such as more recent House elections and to those with very few uncontested races, such as Senate elections.

Consider first an environment with outside spending as follows:

$$\begin{aligned} vote_I = {} & \beta_I \ln d_I + \beta_C \ln d_C + \beta_{I,out} \ln d_{I,out} + \beta_{C,out} \ln d_{C,out} \\ & + \beta_{ten} \ln ten_I + \beta_X X + q_I - q_C + \varepsilon, \end{aligned}$$

where $d_{I,out}$ and $d_{C,out}$ denote outside spending supporting the incumbent and the challenger, respectively. Our approach directly extends to the identification of $\beta_{I,out}$ and $\beta_{C,out}$ because potential endogeneity between $\{d_{I,out}, d_{C,out}\}$ and $\{q_I, q_C\}$ can be controlled for by projecting all of the variables on the predetermined state variables s. Moreover, if there

[17] As we remarked after Proposition 2-2, we can use $P_e(\mathbf{s})$ as a sufficient statistic instead of higher moments of M.

[18] We cannot use elections in which the incumbent experiences an uncontested election *in the future*, because it introduces selections in ε; Experiencing an uncontested election in the future means that the incumbent wins the current election, which implies a high ε value.

exist variables that impact outside groups' spending incentives that are orthogonal to q_I and q_C in a given district, including them as part of the state variable s provides extra source of variation to identify $\beta_{I,out}$ and $\beta_{C,out}$. An example of this type of variable is the number of other races that are predicted to be very close.[19] Note that the sample selection bias can be dealt with in the same way as before even with outside spending.

Consider next an environment in which there are very few uncontested elections. In this case, we cannot invert the policy function of the incumbent in uncontested periods. However, if the researcher has access to additional data on the predicted vote shares, it is possible to extend our approach to this case as well. In Online Appendix 8.7, we describe conditions under which our approach can be modified to estimate the vote share equation and to identify valence terms.

3.2 Identification and Estimation of Challengers' Valence and Components of Utility

We next consider identification and estimation of the challenger's valence, q_C, the standard deviation of the shock in the vote share equation, σ_ε, the cost of raising money, $C_I(\cdot;\theta)$, $C_C(\cdot;\theta)$, $\tilde{C}_I(\cdot;\theta)$ and the consumption value of spending, $H_I(\cdot;\theta)$, $H_C(\cdot;\theta)$, $\tilde{H}_I(\cdot;\theta)$. We parameterize the cost of raising money and the consumption value of spending by θ, where θ is a vector of unknown parameters. We take q_I and the parameters of the vote share equation as given. We also fix the discount factor to 0.9. Because Congressional elections take place every two years, $\delta = 0.9$ corresponds to an annual discount of roughly 0.95. We also normalize the utility from winning, B, to one.[20]

First-Order Conditions The conditions we exploit in our identification and estimation are the candidates' first-order conditions. The first-order conditions associated with the contested incumbent's spending and saving decisions are as follows:

$$\underbrace{\frac{\partial C_I}{\partial d_I}(w_I' + d_I - w_I, q_I;\theta)}_{\text{MC of fund-raising}} = \underbrace{\frac{\beta_I}{\sigma_\varepsilon d_I}\phi(K)\cdot\left(B + \delta\mathbb{E}_{X'|X}[V_I(\mathbf{s}')]\right) + \frac{\partial H_I}{\partial d_I}(d_I;\theta)}_{\text{MB of spending}} \quad (11)$$

[19] If outside groups have budget constraints, the presence of other close races will impact spending in a given election.

[20] Identifying the discount factor in dynamic games is known to be difficult (Magnac and Thesmar 2002). We follow the literature in taking δ as given. Normalizing B to one implies that costs and benefits are measured relative to the utility of winning the election.

$$\underbrace{\frac{\partial C_I}{\partial w'_I}(w'_I + d_I - w_I, q_I; \theta)}_{\text{MC of fund-raising}} = \underbrace{\delta\Phi\left(K\right)\frac{\partial}{\partial w'_I}\mathbb{E}_{X'|X}[V_I(\mathbf{s}')]}_{\text{MB of saving}}, \tag{12}$$

where

$$K = \frac{1}{\sigma_\varepsilon}(\beta_I \ln d_I + \beta_C \ln d_C + q_I - q_C + \beta_{ten} \ln ten_I + \beta_X X). \tag{13}$$

The first expression equates the marginal cost of raising money to the marginal benefit of spending. The marginal benefit consists of an increase in the probability of winning the election multiplied by the continuation value (the first term of the right-hand side of expression(11)) and the incremental consumption value of spending (second term of the right-hand side of (11)). Expression (12) equates the marginal cost of saving to the marginal benefit of saving, which is the incremental value of having more war chest next period. $\Phi(\cdot)$ and $\phi(\cdot)$ are the c.d.f and the p.d.f of the standard normal distribution. These expressions are obtained by substituting $\Pr(vote_I > 0.5)$ in expression (3) using expression (2) and taking derivatives.

Consider the first-order conditions of the incumbents for whom we know the value of q_I, i.e., incumbents who are uncontested at least once. As we discuss below, we can simulate as a function of θ and σ_ε the continuation value $\mathbb{E}_{X'|X}[V_I(\mathbf{s}')]$ and compute its derivative $\frac{\partial}{\partial w'_I}\mathbb{E}_{X'|X}[V_I(\mathbf{s}')]$. This allows us to solve for two values of K using equations (11) and (12) for a given θ and σ_ε. Our identification of θ and σ_ε relies on the restriction that the values of K obtained from these two expressions for each incumbent coincide at the true parameter values.[21] Once the model parameters in (11) and (12) are identified, the value of K for each election is identified, which implies identification of q_C through expression (13). q_C of the challengers who run against incumbents whose q_I is known (i.e., incumbents who experience uncontested elections) are identified at this stage.

The payoff terms of uncontested incumbents and challengers, $\tilde{C}_I$, $\tilde{H}_I$, C_C and H_C are identified analogously by using the first-order conditions of uncontested incumbents whose q_I are known, and those of the challengers whose q_C are identified in the procedure outlined above.

Evaluating the Continuation Value by Simulation We now discuss how to express the continuation value as a function of θ and σ_ε. Our approach is to adapt simulation meth-

[21] To the extent that q_I can be recovered without any error, the first-order conditions must hold with equality for each observation whenever $(d_I, w'_I) > 0$. Of course, in practice, q_I is estimated nonparametrically, and the first-order conditions do not hold exactly at the estimated q_I in finite sample.

ods developed by Hotz, Miller, Sanders and Smith (1994) and Bajari, Benkard and Levin (2007). These simulation methods involve (1) identifying and estimating the transition of the state variables and the equilibrium policy functions nonparametrically; and (2) forward-simulating the value function for a given parameter using the transition probabilities and the policy functions. Importantly, they do not require solving for an equilibrium at each candidate parameter value.

Estimating policy functions requires that we observe all of their arguments. In our application, however, we do not observe q_C, one of the state variables in contested elections. We overcome this issue by modifying the standard procedure. Instead of estimating the policy functions in contested elections, we nonparametrically estimate the distribution of actions and outcomes conditional on the observable states, s, where $\mathbf{s} \equiv \{q_I(\bar{\mathbf{s}}_U), w_I, ten_I, X\}$. More specifically, we estimate the distribution of spending, fund-raising, the probability that the incumbent wins and the probability of retirement as functions of $\mathbf{s}$: $F_{d_I|\mathbf{s}}, F_{fr_I|\mathbf{s}}$, $\Pr(vote_I{>}0.5|\mathbf{s})$, $\lambda(\mathbf{s})$. We also estimate the distribution of incumbent savings conditional on winning, $F_{w'_I|\mathbf{s},\{vote_I>0.5\}}$. The randomness in these variables stems from the random realizations of q_C. We also estimate the transition of exogenous states, $X'|X$. Note that these conditional distributions are identified because all of the variables are observed.

In order to see how to simulate the continuation payoff, consider starting from a period with a given s. We first draw a random variable distributed uniform $\mathrm{U}(0,1)$ to determine whether or not the incumbent retires. If the realization is less than $\lambda(\mathbf{s})$, the incumbent retires and the simulation stops. Otherwise, we draw another random variable distributed $\mathrm{U}(0,1)$ to determine whether or not there is entry. There is entry if the realization is less than $P_e(\mathbf{s})$, where $P_e(\mathbf{s})$ is the probability that there is at least one entrant, estimated in the previous step as a sufficient statistic. Otherwise, there is no entry. Conditional on entry, we draw amount spent and amount raised according to $F_{d_I|\mathbf{s}}(\cdot)$ and $F_{fr_I|\mathbf{s}}(\cdot)$. We also draw a random variable distributed $\mathrm{U}(0,1)$ and compare it to $\Pr(vote_I{>}0.5|\mathbf{s})$ to determine whether or not the incumbent wins. This allows us to simulate a realization of the period utility u_I. If the incumbent loses, the simulation stops. If the incumbent wins, we draw savings according to $F_{w'_I|\mathbf{s},\{vote_I>0.5\}}$ and draw a realization for X'. We then move on to the next period with the new state variables. The case in which there is no entry is analogous.

In this manner, we can draw a sequence of actions and outcomes as well as an associated sequence of flow payoffs which can then be averaged across simulation draws to evaluate $V_I(\mathbf{s})$. The reason we can forward-simulate $V_I(\mathbf{s})$ even though we don't observe q_C is that the incumbent's utility does not depend directly on q_C but only indirectly through actions and outcomes, and that q_C is independent across periods conditional on observables. Moreover, because the period utility is additively separable with respect to actions and outcomes, forward-simulation only requires the marginal, and not joint, distributions of actions and outcomes. Once we simulate $V_I(\mathbf{s})$, we can obtain $\mathbb{E}_{X'|X}[V_I(\mathbf{s})]$ as well as $\frac{\partial}{\partial w_I'}\mathbb{E}_{X'|X}[V_I(\mathbf{s}')]$. Online Appendix 8.4 contains details on the simulation procedure.

Open-Seat Elections To identify the model primitives for open-seat elections, consider the set of open-seat elections in which the eventual winner appears as an incumbent in the estimation procedure outlined above. Under the assumption that candidate valence remains constant over time, the valence measures of these candidates are known. Now consider the first-order condition of these candidates in open-seat elections. The first-order conditions are analogous to expressions (11) and (12). Given that the valence measures of these candidates are known and the primitives of the elections with incumbents are known, the continuation value that appears in the first-order conditions of these candidates ($\mathbb{E}_{X'|X}[V_I(\mathbf{s}')]$ and $\frac{\partial}{\partial w_I'}\mathbb{E}_{X'|X}[V_I(\mathbf{s}')]$) are also known. This means that the primitives for open-seat elections can be recovered from the first-order conditions.

Note that by restricting the estimation sample to elections in which the valence measure of one of the candidates is known, we are selecting the sample partly based on the realization of ε, the error term in the vote share equation.[22] However, given that the candidates choose actions so as to satisfy the first-order conditions before ε is realized, the selection on ε does not bias our estimates.

Recovering Valence for All Candidates We now consider recovering the valence terms for incumbents who were never uncontested and challengers that run against them. Because the vote share equation and the payoff functions of the candidates are known, the four first-order conditions of the candidates in each contested election (two for each candidate) can be used to identify the valance terms of these candidates: the first-order conditions can be considered as a system of equations in q_I and q_C. Similarly, for open-seat elections in which the valence terms of both candidates are yet to be identified, the first-order conditions can be considered as equations in $\mathbf{q_O}$, where $\mathbf{q_O}$ is a 2×1 vector that represents the valence of open-seat candidates. We recover the valence measures of all candidates by solving these first-order conditions for the valence measures. Note that

[22]If the valence measure of one of the candidates is known, it means that the candidate won the open-seat election. Hence, those candidates must have received a favorable value of ε in the election.

first-order conditions of those whose valence is not known at this stage were not used in estimating the parameters of the payoff functions, θ, in the previous stage.

3.3 Estimation

Our estimation closely follows the step-by-step identification procedure described above. In step 1, we estimate the vote share equation using the control function approach. We estimate the expected number of Primary entrants, $\mathbb{E}[M|\mathbf{s}]$, and the probability that a challenger enters to an election, $P_e(\mathbf{s})$. We then estimate the vote share equation by sieve minimum distance estimator (Ai and Chen 2005). In step 2, we estimate candidates' payoffs and the challengers' valence terms by GMM in which we treat the candidates' first-order conditions as well as orthogonality conditions from the vote share equation as moments. To forward-simulate continuation payoffs, we estimate the distribution of actions in contested elections by nonparametric maximum likelihood (Gallant and Nychka 1987). We also estimate the policy functions in uncontested elections as well as the evolution of state variables by regression. We then forward-simulate the continuation payoffs according to the procedure above. In step 3, we estimate parameters of open-seat elections using GMM analogous to the case of contested elections. In step 4, we recover the valence terms for all candidates by GMM in which the candidates' first-order conditions are treated as moments.[23] The details of our estimation strategy are described in Online Appendix 8.6.

4 Data

We obtain the campaign finance data from the Federal Election Commission. The data contain information on the amount of fund-raising, spending and savings of all U.S. House candidates from 1984 to 2002. Data on electoral outcomes and candidate characteristics are obtained from the database of the CQ Press. We obtain demographic characteristics of congressional districts from the Census and the Bureau of Labor Statistics. We also use Presidential election vote shares to create the partisanship measures of each district.

[23] At this stage, we estimate valence measures of all candidates regardless of whether or not their valence measures are recovered in earlier steps. Estimates of valence obtained at this stage may differ from the ones from the previous steps because we can use more data to estimate candidate valence from the first-order conditions (For example, we only use observations after the first uncontested race to recover the valence measures of uncontested incumbents, whereas we can use data from all periods by using the first-order conditions). For the subset of candidates whose valence terms are recovered twice, the average difference between the two estimates of valence is 0.005. We report the valence estimated at this stage in the result section.

	(1)		(2)		(3)		(4)	
	Contested				Uncontested		Open-Seat	
	Incumbent		Challenger					
Spending (d)	418.2	(231.9)	151.3	(188.4)	223.0	(151.9)	429.3	(331.8)
Amount Raised (fr)	442.4	(228.4)	153.3	(190.1)	285.8	(163.2)	438.1	(334.6)
War Chest (w)	76.4	(105.6)	0.5	(3.8)	110.1	(137.1)	0.6	(5.9)
Savings (w')	100.9	(129.4)	2.6	(7.0)	173.6	(172.1)	10.0	(28.5)
Tenure (ten)	5.8	(3.8)	1	(0)	6.5	(4.1)	1	(0)
Vote Share	0.641	(0.086)	0.359	(0.086)	1	(0)	0.5	(0.417)
Sample Size	2,459		2,459		580		356	

Note: Spending, Amount Raised, War Chest and Savings are reported in units of \$1,000. Dollar values are deflated to 1984. Standard Errors are reported in parenthesis.

Table 1: Descriptive Statistics of Incumbents, Challengers and Open-Seat Candidates

Presidential vote shares are obtained from Adler (2003) and POLIDATA.[24]

From the set of regular-cycle House elections, we drop elections in Louisiana, elections in Texas in 1996 that are affected by Supreme Court rulings and elections involving major scandals. We also drop contested elections in which the spending and savings of one of the candidates are zero or very close to zero.[25] Lastly, we drop elections in which one of the candidates either saves or spends more than \$1.2 million (in 1984 dollars).[26] Appendix 8.5 describes in more detail how the data are constructed.

Table 1 reports the summary statistics of the key variables. Dollar values are normalized to 1984 dollars and reported in units of \$1,000. Column (1) corresponds to sample statistics for the incumbents in contested elections and Column (2) corresponds to those for the challengers. In contested elections, incumbents start out with an average war chest of about \$76,400 and raise about \$442,400. The incumbent spends about \$418,200 and saves about \$100,900, on average. The challengers, on the other hand, typically start out with zero war chest and raise about \$153,300, almost all of which is spent. Average incumbent vote share is 64.1%.

[24] https://polidata.org/default.htm

[25] When savings and spending are zero, the first-order conditions of the candidates do not necessarily hold with equality. We drop elections in which one of the candidates spends less than \$5,000 and saves less than \$5,000.

[26] These elections account for 17 and 80 elections, respectively. Unusually large amount of savings are invariably for running for higher offices.

		(1) Democrat		(2) Republican		(3) Open-Seat	
Panel (A)							
	% Unemployed	0.063	(0.024)	0.055	(0.021)	0.059	(0.022)
	Partisanship Index	0.095	(0.510)	-0.191	(0.338)	-0.044	(0.439)
	Party of President	-0.268	(0.964)	-0.097	(0.996)	-0.174	(0.986)
Panel (B)							
	Entry probability	0.802	(0.398)	0.817	(0.387)		
	# Entrants in Primary	1.544	(1.352)	1.542	(1.347)		
	Sample Size	1,670		1,369		356	

Note: Variables in Panel (A) are used as controls in the vote share equation, and those in Panel (B) are used to compute sufficient statistics. Standard errors are reported in parentheses.

Table 2: Characteristics of Congressional Districts

Column (3) of Table 1 corresponds to the sample statistics for the incumbents in uncontested elections. Uncontested incumbents start with an average war chest of $110,100, which is higher than the average war chest of contested incumbents. The average amount of money raised in uncontested periods is about $285,800 and the average amount spent is about $223,000. Incumbents save more in uncontested races ($173,600) than in contested races ($100,900). Column (4) reports the sample statistics for open-seat elections.

Table 2 reports the summary statistics of the characteristics of the Congressional Districts that we include as control variables in the vote share equation (X). Column (1) corresponds to the districts with an incumbent Democrat, Column (2) corresponds to those with an incumbent Republican, and Column (3) corresponds to those of open-seat elections. Partisanship index is a measure of a district's partisanship, obtained as the fitted value of a linear regression in which the regressand is the log difference in the district-level vote shares of the Presidential election and the regressors are demographic characteristics, year fixed effects and state fixed effects. The set of regressors are chosen following Levendusky et. al. (2008). Positive (negative) values of the partisanship index correspond to an expected Presidential vote share above (below) 50% for the Democrats. Appendix 8.5 contains a detailed discussion of how this variable is constructed. Party of President is a dummy variable that equals one if the incumbent president is a Democrat.

5 Specification and Estimation Results

5.1 Specification

Vote Share Equation We specify the vote share equation as follows:

$$\begin{aligned} vote_I = {} & \beta_I \ln d_I + \beta_C \ln d_C + \beta_{ten} \ln ten_I \\ & + \underbrace{\beta_{pt}(pt \times D_I) + \beta_{ue}(ue \times D_I \times D_P)}_{\beta_X X} + q_I - q_C + \varepsilon. \end{aligned}$$

The variable pt is the district's partisanship index constructed from the district level vote shares in Presidential elections as we discussed in Section 4. We interact this variable with D_I which is a variable that is equal to 1 (-1) if the incumbent is a Democrat (Republican). The term $pt \times D_I$ captures predictable cross-sectional variation in the strength of the Democratic candidates across districts.

In order to account for intertemporal variation in the popularity of the parties, we include the term $ue \times D_I \times D_P$. This term captures the effect of retrospective voting. The variable ue is the unemployment rate of the district, which proxies for the current economic environment in the district.[27] Although retrospective voting can take many forms in principle, several studies find that voters express their satisfaction with the current administration by voting for or against the candidate from the president's party (Hibbing and Alford 1981, Stein 1990). For this reason, we interact ue with a variable that indicates whether or not the incumbent and the president are from the same party. Specifically, we let D_P be equal to 1 (-1) if the president is a Democrat (Republican). $D_I \times D_P$ is then equal to one if the candidate and the president are from the same party, and -1, otherwise.

State Variables and Their Transition We assume that ue and pt follow an $AR(1)$ process. We also assume the following process for the President's party, D_P: (1) D_P remains the same next period with probability 0.75 in a presidential election when the president is running for the second term; (2) D_P remains the same with probability 0.5 when the incumbent president is at the end of his second term; and (3) D_P remains the same next period with probability one if the election is a Midterm election. Because the transition of D_P depends on the President's term of office (in his first term or second term)

[27] Unemployment rate is often used as a proxy for the current economic environment, e.g., Ansolabehere et. al. (2014).

and whether or not the election is a Midterm election, these variables are part of the state variables (although they are excluded from the vote share equation). Formally, the vector of state variables X is

$$X = \{pt \times D_I, ue \times D_I \times D_P, 1\{\text{President 1st term}\}, 1\{\text{Midterm}\}\}.$$

Components of the Utility Function We specify the cost of fund-raising and the benefit associated with spending for uncontested incumbents as follows:

$$\tilde{C}_I(fr_I; q_I) = c(q_I)(\ln fr_I)^2,$$
$$\tilde{H}_I(d_I) = \gamma_U \sqrt{\ln d_I},$$

where fr_I denotes the amount raised and $c(\cdot)$ is a decreasing function of q_I. This specification implies that the cost of fund-raising is increasing and convex in $\ln fr_I$ and the benefit associated with spending is increasing and concave in $\ln d_I$. The assumption that $c(\cdot)$ is decreasing in q_I guarantees that q_I is invertible with respect to the actions of the incumbent in uncontested periods as we showed in Proposition 1. In particular, in Online Appendix 8.3 we show that the specific functional form of $\tilde{C}_I$ and $\tilde{H}_I$ allows us to express q_I as a function of a scalar variable $z_U \equiv \frac{fr_I}{\sqrt{\ln d_I} d_I \ln fr_I}$ as $q_I = q_I(z_U)$, where d_I and fr_I are incumbent's spending and fund-raising amount in an uncontested period. Being able to express q_I as a function of a scalar variable, z_U, rather than a vector of all states and actions in the uncontested period, $\bar{\mathbf{s}}_U$, significantly reduces the data requirement for estimation. For estimation, we further specify $c(\cdot)$ as $c(q_I) = c_0 + c_1 \exp(-q_I)$, where $c_0 > 0$ and $c_1 > 0$ are parameters to be estimated. The functional form for $c(q_I)$ ensures that $c(q_I)$ is positive and decreasing.

We specify the cost function of contested incumbents, $C_I(\cdot; q_I)$, and the cost function of challengers, $C_C(\cdot; q_C)$, as follows:

$$C_I(fr_I; q_I) = \eta_I c(q_I)(\ln fr_I)^2$$
$$C_C(fr_C; q_C) = \eta_C c(q_I)(\ln fr_C)^2.$$

	$P_e(\mathbf{s})$		$\mathbb{E}[M\vert\mathbf{s}]$	
Constant	2.934	(1.265)	1.580	(0.638)
$\ln$ War Chest	-0.212	(0.214)	-0.139	(0.089)
$(\ln$War Chest$)^2$	0.006	(0.011)	0.005	(0.005)
$\ln$ Tenure	0.163	(0.108)	0.206	(0.088)
Partisanship Index $\times D_I$	-0.394	(0.116)	-0.281	(0.095)
Unemployment $\times D_I \times D_P$	3.527	(0.848)	2.069	(0.652)
B-Spline of z_U	✓		✓	
Election cycle	✓		✓	

Note: We take 7 knots corresponding to $(1/8,\cdots,7/8)$ quantiles of z_U for the B-Spline of z_U. Election cycle corresponds to a complete interaction of dummy variables $1\{\text{President 1st term}\}$ and $1\{\text{Midterm}\}$. Standard errors are reported in parentheses.

Table 3: Estimates of $P_e(s)$ and $\mathbb{E}[M|s]$

We specify the benefit of spending for contested incumbents and challengers as follows:[28]

$$H_I(d) = H_C(d) = \gamma\sqrt{\ln d}.$$

We assume that the costs of fund-raising and the benefit from spending for open-seat candidates are the same as those of challengers running against incumbents. We specify the retirement probability, $\lambda(\mathbf{s})$, as a nonparametric function of ten_I.

5.2 Parameter Estimates

Estimates of $P_e(\cdot)$ and $\mathbb{E}[M|\mathbf{s}]$ We first report our estimates of challenger entry probability, $P_e(\mathbf{s})$, and the average number of entrants, $\mathbb{E}[M|\mathbf{s}]$. Estimates of $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ are used to control for the sample selection problem in expression (1$''$). Both $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ are functions of $\mathbf{s} = (q_I, w_I, ten_I, X)$. Because q_I can be expressed as a function of the incumbent's actions in uncontested periods as $q_I = q_I(z_U)$ ($z_U \equiv \frac{fr_I}{\sqrt{\ln d_I} d_I \ln fr_I}$), we estimate $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ as functions of (z_U, w_I, ten_I, X). Note that all of the arguments are observed.

[28]We assume $H_C(\cdot) = H_I(\cdot)$ because it is difficult to separately estimate $H_C(\cdot)$ from the challengers' fund-raising cost, $C_C(\cdot)$, in our sample. The difficulty arises because (i) the probability of winning the election is very small for many challengers and (ii) many challengers save nothing, i.e., $w'_C = 0$. When the winning probability is very small and $w'_C = 0$, the only binding restriction becomes $C'_C(d_C) = H'_C(d_C)$, which results in a trivial solution that both C_C and H_C are constant (and hence the derivatives are zero).

In principle, $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ should be estimated fully nonparametrically as functions of the state variables because they are equilibrium objects. Given the moderate sample size, however, we estimate P_e by a Probit model and $\mathbb{E}[M|\mathbf{s}]$ by a linear regression. We use as regressors $\ln w_I$, $(\ln w_I)^2$, $\ln ten_I$, X, and B-spline bases of z_U. Online Appendix 8.6 contains further details on the estimation of $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ as well as those of other results in this section.

Table 3 reports the results. The first column of the table reports our estimate of P_e. We find that a higher incumbent war chest is associated with a lower entry probability, and a longer tenure is associated with a higher entry probability, although the estimates are not statistically significant. We find that the unemployment rate and the district partisanship index have statistically and economically significant effects on the entry probability. On average, a one standard deviation increase in the unemployment rate is associated with about a 2.4 percentage-point increase in the entry probability if the incumbent is from the same party as the President, and a one standard deviation change in the district partisanship in the incumbent's favor is associated with a 5.8 percentage-point decrease in the probability of entry. The second column of Table 3 reports the estimated coefficients for $\mathbb{E}[M|\mathbf{s}]$. The sign and significance of the coefficients are similar to those for P_e.

The fact that variables such as the partisanship index and unemployment rate affect $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ suggests that these variables indirectly affect the valence of the general election challenger, q_C, through the challengers' endogenous entry decisions. These results suggest the importance of accounting for sample selection of q_C.

Estimates of Vote Share Equation In the first column of Table 4, we report our estimates of the vote share equation. The parameters are estimated by applying a sieve minimum distance estimator of Ai and Chen (2005) to expression ($1''$). Our point estimates of β_I and β_C are 0.025 and -0.025, respectively, which imply that a standard deviation increase in the spending of the incumbent increases the incumbent vote share by about 1.4 percentage points, while a standard deviation increase in the challengers' spending decreases the incumbent vote share by about 3.8 percentage points.[29] We find that the impact of spending on the vote share is smaller in open-seat elections, although the standard error is quite large. The estimate of β_O is 0.013 which implies that a standard deviation increase in the spending of an open-seat candidate increases the vote share of that candidate

[29]This asymmetry is due to the concavity of the log function and the fact that the mean spending of challengers is lower than that of incumbents.

by about 1.1 percentage points.

We also find that the partisanship index has a large positive effect on the vote share. The estimated coefficient is 0.054, which implies that a standard deviation change in the partisanship index in the incumbent's favor leads to an increase in the incumbent vote share by 2.3 percentage points. Our estimates of the coefficients on the incumbent tenure, β_{ten}, and that on the unemployement rate, β_{ue}, are small and statistically insignificant. Our estimate of σ_{ε}, the standard error of the error term in the vote share equation, is 0.041.

	Control function		OLS	
β_I	0.025	(0.014)	-0.015	(0.004)
β_C	-0.025	(0.011)	-0.029	(0.002)
β_O	0.013	(0.038)		
β_{ten}	0.001	(0.009)	-0.014	(0.006)
β_{pt}	0.054	(0.013)	0.062	(0.007)
β_{ue}	-0.073	(0.058)	-0.096	(0.044)
σ_{ε}	0.041	(0.009)	0.063	(0.007)

Note: First column corresponds to the estimates obtained using the approach discussed in Section 3. Second column corresponds to OLS estimates. Standard errors are reported in parentheses. The standard errors of the estimates reported in the first column are reported based on 500 sets of bootstrap subsample.

Table 4: Parameter Estimates of the Vote Share Equation

In the second column of Table 4, we report the OLS estimates of the vote share equation for elections with incumbents for comparison. The error term of the OLS regression corresponds to $q_I - q_C + \varepsilon$. If incumbents choose a higher d_I against stronger challengers (i.e., q_C and d_I are positively correlated), β_I will be downward-biased in the OLS estimates. Indeed, we find that the OLS estimate of the coefficient on incumbent spending is negative and statistically significant. Similarly, if an electoral environment that is favorable to the incumbent deters strong challengers from entering, the coefficient on the partisanship index and that on the unemployment rate will be biased away from zero. A comparison of the coefficients in the two columns suggests that this is the case.

Estimates of $C(\cdot)$ and $H(\cdot)$ Table 5 reports the estimated parameters of the candidates' cost of fund-raising, $C(\cdot)$, and the personal benefit of spending, $H(\cdot)$. Panel (A) of

$C(\cdot)$			$H(\cdot)$		
η_I	0.008	(0.002)	γ	0.131	(0.063)
η_C	0.007	(0.006)	γ_U	0.101	(0.045)
η_U	0.003	(0.002)			
η_q	0.102	(0.069)			

Note: Standard errors are calculated based on 500 sets of bootstrap subsample and reported in parentheses.

Table 5: Parameter Estimates of Fund-Raising Cost and Benefit from Spending

Figure 2 illustrates the shape of the estimated cost function when the valence measures are set to their means.[30] Panel (B) of Figure 2 illustrates the shape of the benefit function. The vertical axis of the panels corresponds to the cost of fund-raising and the benefit of spending measured relative to the utility from winning, which is normalized to 1. The horizontal axes correspond to the log amount raised and log amount spent. We find that incumbents and challengers face a similar cost function in contested elections. The difference between η_I and η_C is not statistically significant.

5.3 Estimates of Candidate Valence

We now discuss our estimates of candidate valence measures. The top panel of Figure 3 corresponds to the histogram of the estimated valence measures of the incumbents and the middle panel corresponds to that of challengers that run against incumbents.[31] We find that, on average, the valence measures of the incumbents are about 0.094 (9.4 percentage points) higher than those of the challengers, implying that the differences in candidate valence translates to a 9.4 percentage point vote-share advantage for the incumbents. To put this number in perspective, the incumbent won with less than 59.4 percent of the vote share in about 29.5 percent of the elections in our sample. We also find a relatively small dispersion of valence measures among incumbents. The inter-quartile range of incumbent valence is about 2.9 percentage points. On the other hand, the valence measures of the challengers are more dispersed. Our finding that there is a longer tail of low-valence challengers is

[30]We use $q = 0.0134$, 0.0107 and -0.0801 for contested incumbents, uncontested incumbents, and challengers in contested elections, respectively.

[31]If a candidate competes in an election as a challenger and subsequently becomes an incumbent, the valence measure of the candidate is included in both panels. Similarly, candidates that compete in open-seat elections who later become incumbents appear twice in Figure 3, in the top panel and in the bottom panel.

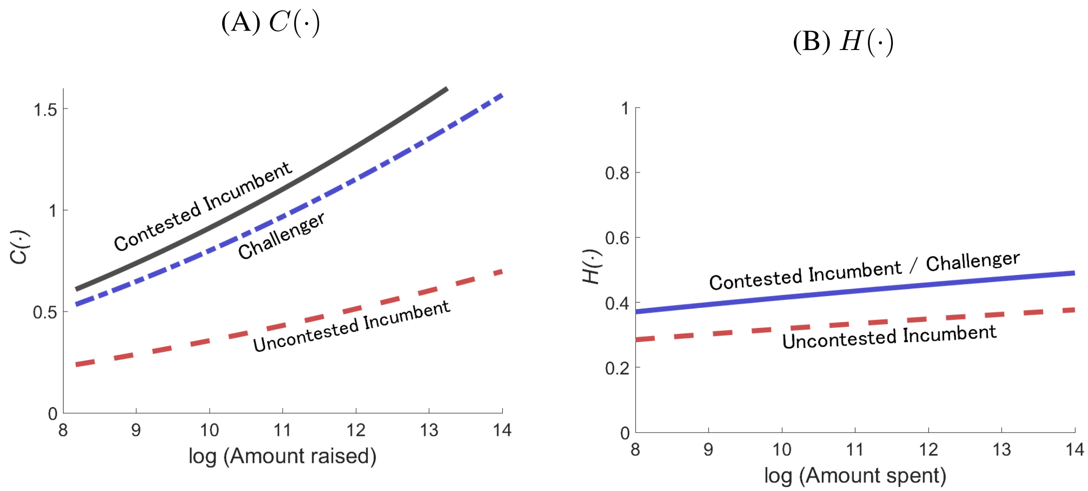

Figure 2: Cost of Fund-Raising and Benefit of Spending

Note: In Panel (A), the horizontal axis corresponds to the log amount raised and the vertical axis corresponds to the cost of raising money relative to the utility of winning. We evaluate $C(\cdot)$ at $q = 0.0134$ for contested incumbents, 0.0107 for uncontested incumbents and -0.0801 for challengers in contested elections. In Panel (B), horizontal axis corresponds to the log amount spent and the vertical axis corresponds to the benefit of spending money measured relative to the utility from winning.

consistent with the fact that incumbents are selected partly by valence.

The bottom panel of Figure 3 plots the histogram of the estimated valence measures for open-seat candidates. We find that the upper tail of the distribution of the open-seat challengers resembles that of the incumbents. However, a substantial fraction of low valence open-seat challengers increases the dispersion of the distribution. The average valence measure of open-seat challengers is about 3.8 percentage points lower than those of the incumbents, and about 5.6 percentage points higher than those of challengers that run against incumbents. Our finding suggests that open-seat challengers are on average stronger than challengers that run against incumbents.

Valence of Winners and Losers, Democrats and Republicans We now report the distribution of candidate valence by whether or not the candidate wins the election, and by the party of the candidate. Panel (A) of Figure 4 illustrates the valence measure of incumbents, challengers, and open-seat candidates by whether or not the candidate wins the election. The gray bars correspond to the winners and the uncolored bars correspond to the losers of the election. For incumbents, we find that the valence distribution of the

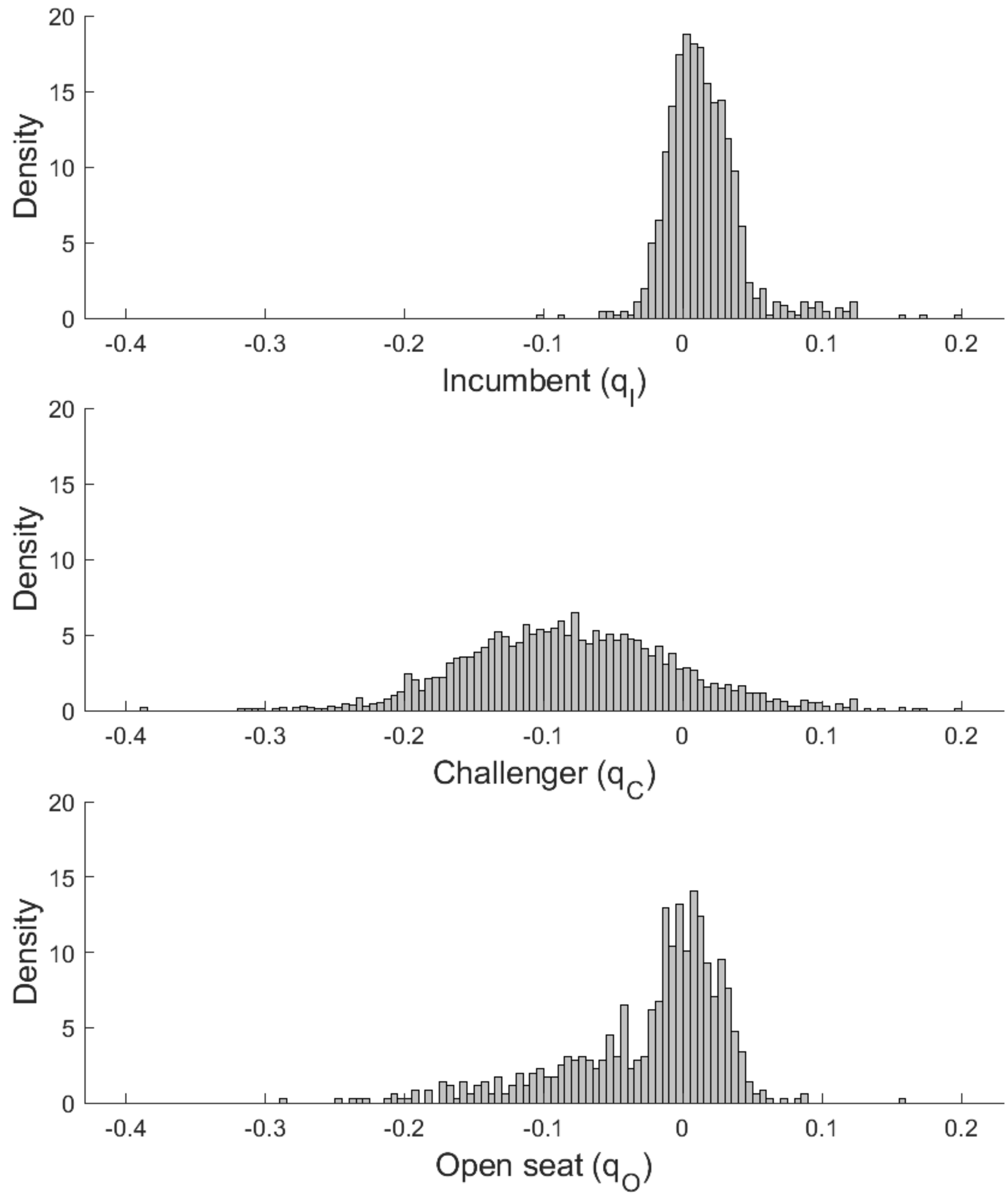

Figure 3: Distribution of Candidate Valence

Note: The top, middle and bottom panels correspond to the histogram of valence measures of incumbents, challengers running against incumbents and open-seat candidates, respectively. The valence measures are scaled in the unit of vote shares.

winners and the losers are similar, although the mean valence is slightly higher for winners (0.0143) than for losers (0.0130). For challengers and open-seat candidates, we find that the valence measures of the winners are much higher than those of the losers. The average valence of challengers that win is 0.049, while the average valence of challengers that lose

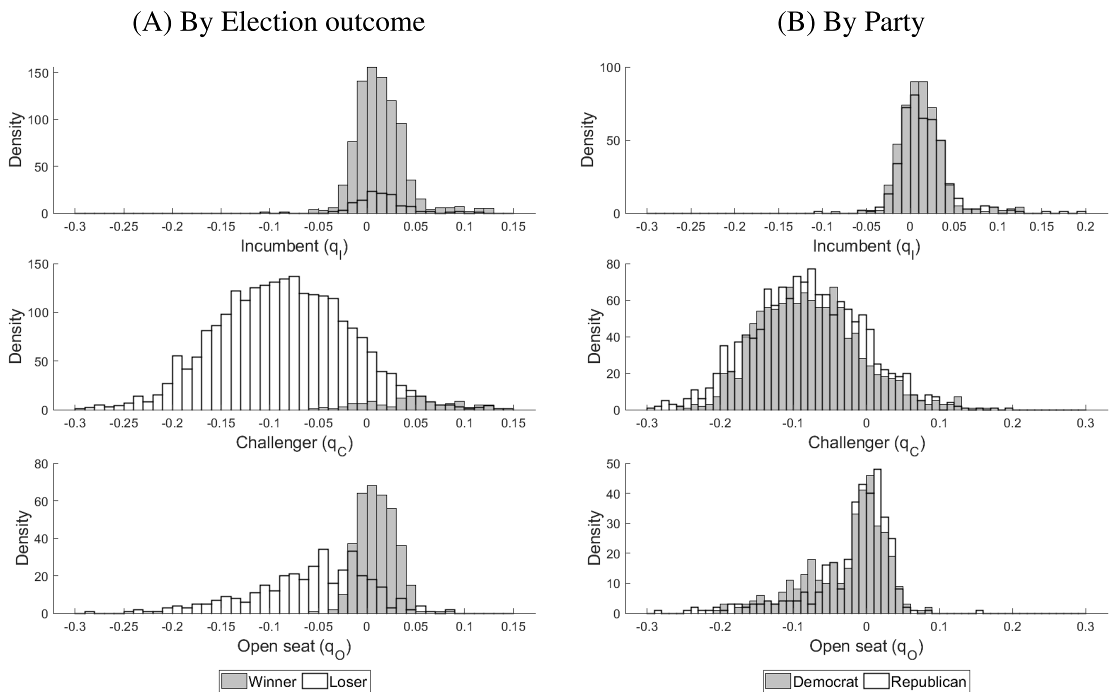

Figure 4: Distribution of Candidate Valence conditional on party and election outcome

Note: Panel (A) and (B) display the distribution of valence measures conditional on the election outcome and the party of the candidate, respectively. The top, middle and bottom panels correspond to the histogram of valence measures of incumbents, challengers running against incumbents and open-seat candidates, respectively. The valence measures are scaled in the unit of vote shares.

is -0.087. The average valence of open-seat candidates that win is 0.011 while the average for those that lose is -0.059. These findings reflect the fact that candidates are selected partly by valence.

Panel (B) of Figure 4 illustrates the valence measures broken down by party. The gray bars correspond to the Democrats and the uncolored bars correspond to the Republicans. We do not find any significant differences in the distribution of valence between the parties.

Comparison with Existing Valence Measures In order to assess the validity of our measure of candidate valence, we compare our measure with an existing one constructed by Maestas and Rugeley (2008). In Maestas and Rugeley (2008), the authors construct four dummy variables that capture the seriousness of challengers that run for House seats between 1992 and 2000. The dummies are constructed based on observed characteristics of the candidates, such as previous political experience and extent of personal investment

		Serious 25	Serious 50	Serious 75	Serious 90
Spearman's ρ					
	q_C	0.115 [0.000]	0.241 [0.000]	0.315 [0.000]	0.358 [0.000]
Regression					
	Const.	0.952 (0.008)	0.855 (0.013)	0.704 (0.015)	0.605 (0.015)
	q_C	0.446 (0.088)	1.396(0.137)	2.111 (0.159)	2.404 (0.159)
Sample size		1699	1699	1699	1699

Note: We use challengers that run for election between 1992 and 2000. We report the Spearman's rank correlation coefficient and the regression coefficients. We report the p-values (in square brackets) for the Spearman's rank correlation coefficient and standard errors (in round brackets) for the regression.

Table 6: Correlation with Seriousness Measure of Maestas and Rugeley (2008)

in the campaigns. Because their measures are specifically aimed at capturing factors that affect a candidate's probability of winning, their measures serve as good benchmarks of comparison.

The first row of Table 6 reports the Spearman's rank correlation coefficient between our measure and each of the four measures of seriousness in Maestas and Rugeley (2008). We report the p-values in square brackets. In all four columns, we find that the rank correlation coefficients are positive and statistically significant. The second row of Table 6 reports the results from linear regressions in which we regress each of the four measures of seriousness on our measure of valence. We find that the coefficient on q_C is positive and statistically significant. These results suggest that our measure captures an important aspect of candidate electability.

6 Source of Incumbency Advantage in U.S. House Elections

Our measures of candidate valence can be potentially useful in studying a wide variety of substantive questions in political economy. In order to illustrate their usefulness, we use the estimated valence measures to study incumbency advantage in U.S. House elections, a topic studied extensively in the literature. Our approach follows that of Lee (2008), who offers perhaps one of the cleanest estimates of the incumbency advantage using a regression discontinuity (RD) design. Lee (2008) compares the Democrat's vote share in period $t+1$ between districts in which the Democrats marginally won in period t and those in which the Democrats marginally lost in period t. Formally, Lee (2008) defines incumbency advantage as follows:

$$\lim_{\epsilon \to +0} \mathbb{E}[vote_{Dem,t+1} | vote_{Dem,t} = 0.5 + \epsilon] - \lim_{\epsilon \to +0} \mathbb{E}[vote_{Dem,t+1} | vote_{Dem,t} = 0.5 - \epsilon], \quad (14)$$

where $vote_{Dem,\tau}$ is the Democrat's vote share in period τ. The RD estimate identifies the extra vote shares that a party gains from fielding a marginal incumbent (against an average challenger from the other party), relative to the case in which the party fields an average challenger (against the rival party's marginal incumbent).

Using the estimated measures of candidate valence and the parameters of the vote share equation, we study the source of the incumbency advantage. Specifically, we consider decomposing the incumbency advantage into a valence effect, a spending effect and a tenure effect. The valence effect is the difference in the valence between marginal winners and average challengers that are fielded against the incumbents. The spending effect is the differential propensity to spend money between marginal incumbents and average challengers. The tenure effect is the differences in tenure: a marginal incumbent has typically served several terms in office at the time of the election in period $t+1$. Identifying the sources of incumbency advantage is important for an informed discussion on ways to reduce the incumbency advantage and increase political competition (e.g. subsidizing challengers' campaigns).

We first estimate the incumbent's overall vote share advantage for our sample by applying expression (14) to our data. Column (1) of Table 7 reports the results.[32] We find that the RD estimate of the incumbency advantage is 10.2 percentage points.[33] Figure 5 shows the binned scatter plot of the Democratic vote share in period $t+1$ against the Democratic vote share in period t.

We now study how much of the incumbency advantage is explained by differences in candidate valence, spending, and tenure. To do so, we estimate the same RD regression as

[32] We use the bias-correction estimator proposed by Calonico et. al. (2014) for all of our RD estimates.

[33] This result is reasonably close to the Lee's original result, which is around 8.0 percentage points. The difference of 2.0 percentage points is likely to reflect the fact that we only use elections from 1984, whereas Lee's data include elections from the 1950s. There is evidence that incumbency advantage seems to be increasing over time (see, e.g., Gelman and King (1990)).

	(1) Vote share	Valence			Spending			(8) Log-Tenure
		(2) Dem	(3) Rep	(4) Total	(5) Dem	(6) Rep	(7) Total	
Estimate	0.102	0.042	-0.017	0.068	0.563	-0.801	1.396	1.947
	(0.017)	(0.010)	(0.014)	(0.013)	(0.176)	(0.164)	(0.185)	(0.149)
In Vote share	0.102			0.068			0.030	0.001
Bandwidth	0.052	0.062	0.042	0.060	0.057	0.060	0.072	0.054
Obs	1853	1853	1853	1853	1853	1853	1853	1853

Note: The sample of elections used for this regression includes all election pairs between period t and $t+1$ in which period $t+1$ is not an uncontested election. The total effect in Columns (4) is estimated with the dependent variable being $q_{Dem,t+1} - q_{Rep,t+1}$ for the case of $vote_{Dem,t} > 0.5$, and $q_{Rep,t+1} - q_{Dem,t+1}$ otherwise. The total effect of spending in Column (7) and the effect on Log-Tenure in Column (8) are estimated analogously. We use a uniform kernel to estimate these RD regressions. Standard errors are reported in parentheses.

Table 7: RD Estimates with respect to Democrat's Two-Party Vote Share in the Previous Election

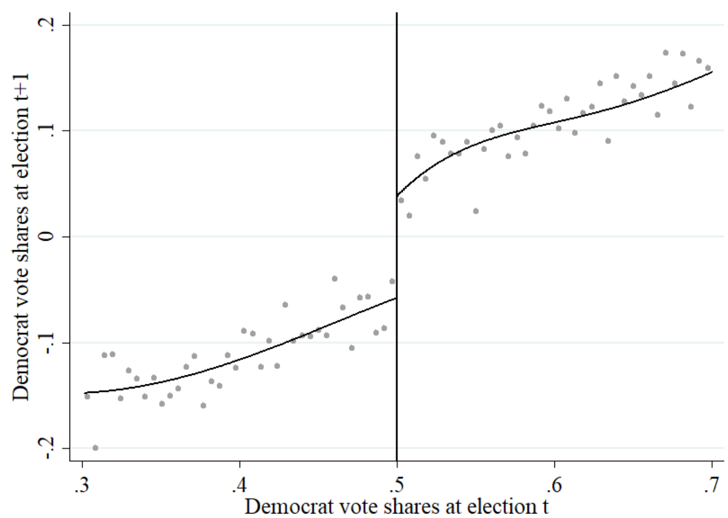

Figure 5: Binned Scatter Plot of Democratic Vote Share

Note: The figure plots the vote share of the Democratic candidates at $t+1$ on the vertical axis and the Democratic candidate's vote share in period t on the horizontal axis. The curves in the figure correspond to a fourth-order polynomial approximation of the conditional expectation.

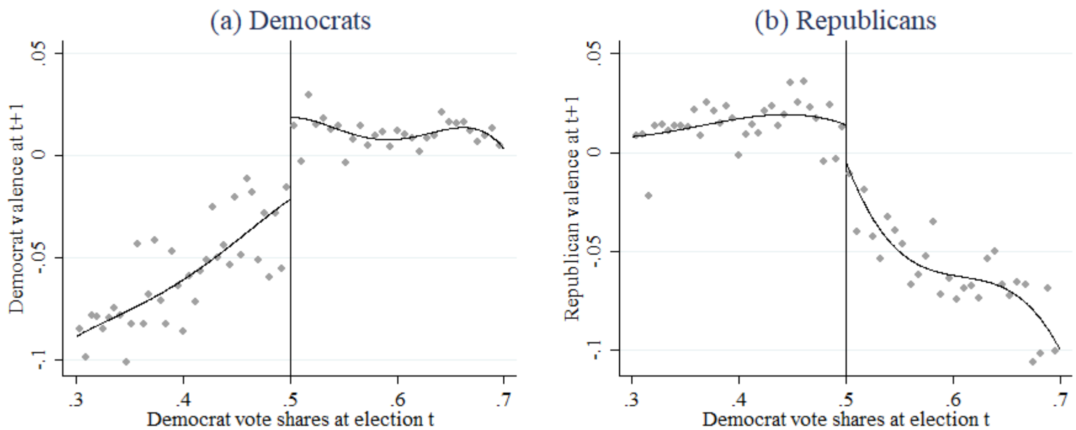

Figure 6: Binned Scatter Plot of Candidate Valence

Note: Panel (A) is the binned scatter plot of the valence of Democratic candidates at $t+1$ and Panel (B) is that of the Republican candidates at $t+1$. The horizontal axis for both panels is the Democratic candidate's vote share in period t. The curves in the figure correspond to a fourth-order polynomial approximation of the conditional expectation.

expression (14), but replacing the outcome variable with valence, spending and tenure of the candidates. Columns (2) through (8) of Table 7 report the results.

Columns (2) through (4) of Table 7 report the RD estimates for candidate valence. Column (2) corresponds to the case in which we take the outcome variable to be the valence of the Democratic candidate in period $t+1$. The estimate implies that a Democratic candidate who marginally wins in period t (and hence is an incumbent in period $t+1$) has higher valence measure than an average Democratic candidate that is fielded as a challenger in period $t+1$ by about 4.2 points. Column (3) reports the corresponding RD estimate for the Republicans. The estimate implies that an average Republican challenger has lower valence than a marginal Republican incumbent by about 1.7 points. Thus, a narrow Democratic win in period t relative to a narrow Democratic loss implies a stronger Democratic candidate as well as a weaker Republican candidate in $t+1$. Column (4) reports the combined effect, which is estimated to be about 6.8 percentage points.[34] Panel (A) of Figure 6 is the binned scatter plot of the valence of the Democrats in period $t+1$ and Panel (B) is that of the Republicans in period $t+1$. In both panels, the horizontal axis is the Democratic

[34] The total effect in Columns (4) is estimated by an RD regression with the dependent variable being $q_{Dem,t+1} - q_{Rep,t+1}$ for the case of $vote_{Dem,t} > 0.5$, and $q_{Rep,t+1} - q_{Dem,t+1}$ otherwise. In the absence of sampling error, the combined effect should be exactly equal to the effect for the Democratic candidates minus the effect for the Republican candidates.

party's two-party vote share in period t.

Columns (5) through (7) of Table 7 report the RD estimates for spending. For Column (5), we take the outcome variable in the RD regression to be the log spending of the Democratic candidate in period $t+1$. The coefficient estimate implies that the log spending of a marginal Democratic incumbent is higher than the log spending of an average Democratic challenger by about 0.56 points. Similarly, the coefficient estimate in Column (6) implies that the log spending of an average Republican challenger is lower than a marginal Republican incumbent by about 0.80 points. The estimates correspond to about a 200,000 dollar difference in spending for the Democrats and 310,000 dollar difference for the Republicans. Column (7) of Table 7 reports the combined effect, which is estimated to be 1.40 points in terms of log spending, or about 480,000 dollars. Given our coefficient estimate of spending in the vote share equation, differences in spending amount to an incumbency advantage of about 3.0 percentage points in terms of vote share. Panel (A) of Figure 7 is the binned scatter plot of spending by the Democratic candidate in period $t+1$ and Panel (B) is the corresponding plot for the spending of the Republicans.

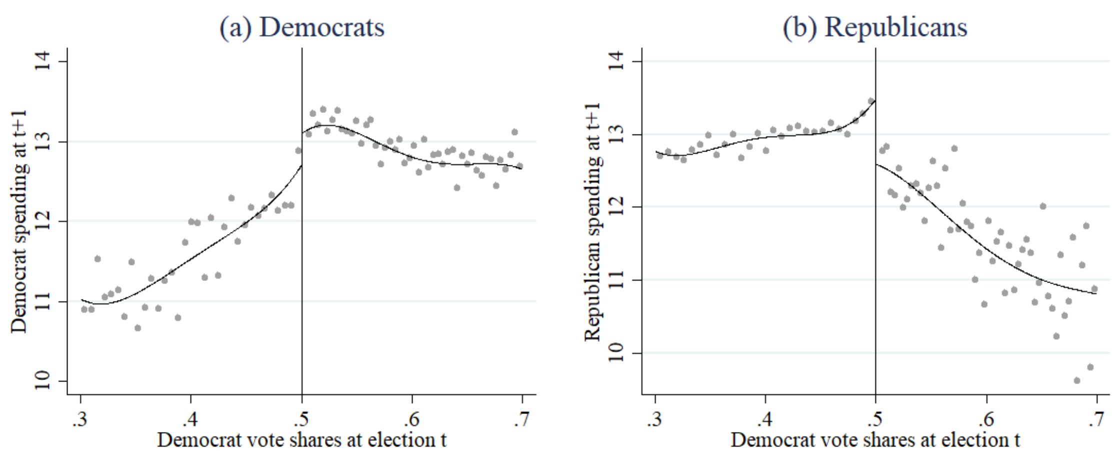

Figure 7: Binned Scatter Plot of Candidate Spending

Note: The left panel is the binned scatter plot of the log spending of Democratic candidates at $t+1$ and the right panel is that of the Republican candidates at $t+1$. The horizontal axis for both panels is the Democratic candidate's vote share in period t. The curves in the figure correspond to a fourth-order polynomial approximation of the conditional expectation.

Lastly, we consider the component of the incumbency advantage that is attributable to the differences in the tenure of the candidates. The RD estimate of the log tenure of the Democrats is 1.11 points, or 3.15 terms and the estimate for the Republicans is -0.83 points, or -1.52 terms. These estimates imply that a marginal Democratic winner has served about 3.15 terms in office by the election in period $t + 1$ and the marginal Republican winner has served about 1.52 terms in office. Column (8) of Table 7 reports the combined effect on log tenure, which is 1.95 points, or 4.69 terms. Given that our coefficient estimate on tenure in the vote share equation is small (0.001), the differences in tenure between the incumbent and the challenger translates to an incumbency advantage of about 0.1 percentage points in terms of vote share.

To summarize, we find that the incumbency advantage that results from differences in candidate valence accounts for about 6.8 percentage points in terms of vote share. Differences in candidate spending accounts for about 3.0 percentage points and differences in the log experience of the candidates account for about 0.1 percentage points. Our results suggest that differences in candidate valence account for a substantial component of the incumbency advantage. This in turn suggests that policy interventions designed to reduce incumbency advantage through the spending channel, such as subsidizing challengers' campaigns, may have limited effectiveness.

7 Conclusion

This study is a first attempt at recovering the valence of candidates from observations of vote shares and candidates' campaign finance decisions. Although candidate valence plays an important role in many theoretical work of political competition, empirical measures of valence has been mostly lacking. We think that the methods developed in this paper can serve as a starting point for testing and estimating models of political competition in which valence plays an important role.

References

[1] Ackerberg, N., Caves, K. & Frazer, G. (2006), "Identification Properties of Recent Production Function Estimators," *Econometrica* 83(6), 2411–2451.

[2] Adams, J. & Merrill, S. III (2008), "Candidate and Party Strategies in Two-Stage Elections Beginning with a Primary" *American Journal of Political Science* 52(2), 344–359.

[3] Adler, S. E. (2003), “Congressional District Data File, 1984-2000.” University of Colorado, Boulder, CO.

[4] Ai, C. & Chen, X. (2003), “Efficient Estimation of Models with Conditional Moment Restrictions Containing Unknown Functions,” *Econometrica* 71(6), 1795–1843.

[5] Ansolabehere, S. & Snyder, J. M. Jr. (2004), “Using Term Limits to Estimate Incumbency Advantages When Officeholders Retire Strategically,” *Legislative Studies Quarterly* 29(4), 487–515.

[6] Ansolabehere, S., Snyder, J. M. Jr. & Stewart, C. III (2000), “Old Voters, New Voters, and the Personal Vote: Using Redistricting to Measure the Incumbency Advantage,” *American Journal of Political Science* 44(1),17–34.

[7] Ansolabehere, S., Meredith, M & Snowberg, E. (2014), “Mecro-Economics Voting: Local Information and Micro-Perceptions of the Macro-Economy,” *Economics and Politics* 26(3), 380–410.

[8] Aragones, E. & Palfrey, T. R. (2004), “The Effect of Candidate Quality on Electoral Equilibrium: An Experimental Study,” *American Political Science Review* 98(1), 77–90.

[9] Bajari, P. C., Benkard, L. & Levin, J. (2007), “Estimating Dynamic Models of Imperfect Competition,” *Econometrica* 75(5), 1331–1370.

[10] Calonico, S., Cattaneo, M. & Titiunik, R. (2014). “Robust Nnonparametric Confidence Intervals for Regression-Discontinuity Designs,”*Econometrica* 82(6) 2295–2326.

[11] Canen, N., Kendall, C. and Trebbi, F. (2018), “Unbundling Polarization,” *Working Paper*.

[12] Carter, J. & Patty, J. W. (2015) “Valence and Campaigns,” *American Journal of Political Science* 59(4), 825–840.

[13] Cox, G. W. & Thies, M. F. (2000) “How Much Does Money Matter?: ”Buying” Votes in Japan, 1967-1990,” *Comparative Political Studies* 33(1), 37–57.

[14] da Silveira, B., & de Mello, J. (2011) “Campaign Advertising and Election Outcomes: Quasi-natural Experiment Evidence from Gubernatorial Elections in Brazil,” *Review of Economic Studies* 78(2), 590–612.

[15] Diermeier, D., Keane, M. & Merlo, A. (2005), “Political Economy Model of Congressional Careers,” *American Economic Review* 95(1), 347–373.

[16] Diermeier, D., Eraslan, H. & Merlo, A. (2003). “A Structural Model of Government Formation,” *Econometrica* 71(1), 27–70.

[17] Ericson, R. S. & Pakes, A. (2001), “Markov-Perfect Industry Dynamics: A Framework for Empirical Work,” *Review of Economic Studies* 62(1), 53–82.

[18] Erikson, R. S. (1971) “The Advantage of Incumbency in Congressional Elections,” *Polity*, 3, 3, 395–405.

[19] Erikson, Robert S, and Thomas R. Palfrey (2000) “Equilibria in Campaign Spending Games: Theory and Data,” *American Political Science Review*, 94, 3, 595–609.

[20] Gallant, R. A. & Nychka, D. W. (1987), “Semi-Nonparametric Maximum Likelihood Estimation,” *Econometrica* 55(2), 363–390.

[21] Garcia-Jimeno, C. & Yildirim, P. (2018), “Matching Pennies on the Campaign Trail: An Empirical Study of Senate Elections and Media Coverage,” *Working Paper*.

[22] Gelman, A. & King, G. (1990) “Estimating Incumbency Advantage without Bias,” *American Journal of Political Science* 34(4), 1142–1164.

[23] Gerber, A. (1998) “Estimating the Effect of Campaign Spending on Senate Election Outcomes Using Instrumental Variables,” *American Political Science Review* 92(2), 401–411.

[24] Gandhi, A., Navarro, S. & Rivers, D. A. (2020) “On the Identification of Gross Output Production Functions,” *Journal of Political Economy* 128(8), 2973-3016.

[25] Gowrisankaran, G,, Mitchell, M. & Moro, A. (2008) “Electoral Design and Voter Welfare from the U.S. Senate: Evidence from a Dynamic Selection Model,” *Review of Economic Dynamics* 11(1), 1–17.

[26] Green, D. & Krasno, J. (1988) "Salvation for the Spendthrift Incumbent: Reestimating the Effects of Campaign Spending in House Elections," *American Journal of Political Science* 32(4), 884–907.

[27] Groseclose, T. (2001) "A Model of Candidate Location When One Candidate Has a Valence Advantage," *American Journal of Political Science* 45(4), 862–886.

[28] Hibing, J. R. & Alford, J. R. (1981), "The Electoral Impact of Economic Conditions: Who is Held Responsible?," *American Journal of Political Science* 25(3), 423–439.

[29] Hotz, J. V., Miller, R., Sanders, S. & Smith, J. (1994), "A Simulation Estimator for Dynamic Models of Discrete Choice," *Review of Economic Studies* 61(2), 265–289.

[30] Iaryczower, M., Moctezuma, G. L. & Meirowitz, A. (2018), "Career Concerns and the Dynamics of Electoral Accountability," Working paper.

[31] Jacobson, G. C. (1978) "The Effects of Campaign Spending in Congressional Elections," *American Political Science Review* 72(2), 469–491.

[32] Lee, D. S. (2008) "Randomized Experiments from Non-Random Selection in U.S. House Elections," *Journal of Econometrics* 142(2), 675–697.

[33] Levendusky, M. S., Pope, J. C. & Jackman, S. D. (2008) "Measuring District-Level Partisanship with Implications for the Analysis of U.S. Elections," *The Journal of Politics* 70(3), 736–753.

[34] Levinsohn, J. & Petrin, A. (2003) "Estimating Production Function Using Inputs to Control for Unobservables," *Review of Economic Studies* 70(2), 317–341.

[35] Levitt, S. D. (1994) "Using Repear Challengers to Estimate the Effect of Campaign Spending on Election Outcomes in the U.S. House," *Journal of Political Economy* 102(4), 777-798.

[36] Levitt, S. D. & Wolfram, C. D. (1997) "Decomposing the Sources of Incumbency Advantage in the U.S. House," *Legislative Studies Quarterly* 22(1), 45–60.

[37] Lim, C. S. H., & Yurukoglu, A. (2018) "Dynamic Natural Monopoly Regulation: Time Inconsistency, Moral Hazard, and Political Environments," *Journal of Political Economy* 126(1), 263–312.

[38] Maestas, C. D. & Rugeley, C. R. (2008), "Assessing the Experience Bonus Through Examining Strategic Entry, Candidate Quality, and Campaign Receipts in U.S. House Elections," *American Journal of Political Science* 52(3), 520–535.

[39] Magnac, T. & Thesmar, D. (2002), "Identifying Dynamic Discrete Decision Processes," *Econometrica* 70(2), 801–816.

[40] Maskin, E. & Tirole, J. (1988). "A theory of dynamic oligopoly, I: Overview and quantity competition with large fixed costs," *Econometrica* 56(3), 549–569.

[41] Merlo, A. (1997). "Bargaining over Governments in a Stochastic Environment," *Journal of Political Economy* 105(1), 101–131.

[42] Olley, S. & Pakes, A. (1996) "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica* 64(6), 1263–1297.

[43] Poole, K. and Rosenthal, H. (1985). "A Spatial Model For Legislative Roll Call Analysis,"*American Journal of Political Science* 29(2), 357-384.

[44] Serra, G. (2011), "Why Primaries? The Party's Tradeoff between Policy and Valence," *Journal of Theoretical Politics* 23(1), 21–51.

[45] Sieg, H. & Yoon, C. (2017), "Estimating Dynamic Games of Electoral Competition to Evaluate Term Limits in US Gubernatorial Elections," *American Economic Review* 107(7), 1824–1857.

[46] Snyder, J. M. & Ting, M. M. (2011), "Electoral Selection with Parties and Primaries," *American Journal of Political Science* 55(4), 782–796.

[47] Stein, R. M. (1990), "Voting for Governor and U.S. Senator: The Electoral Consequences of Federalism," *The Journal of Politics* 52(1), 29–53.

[48] Stone, J. W., Futon, S. A., Maestas, C. D. & Maisel, L. S. (2010), "Incumbency Reconsidered: Prospects, Strategic Retirement, and Incumbent Quality in U.S. House Elections," *The Journal of Politics* 72(1), 178-190.

[49] Stone, J. W. & Simas, E. N. (2010), "Candidate Valence and Ideological Positions in U.S. House Elections," *American Journal of Political Science* 54(2), 371-388.

[50] Stratmann, T. (2005), "Some talk: Money in politics. A (partial) review of the literature," *Public Choice* 124,135–156.

[51] United States General Accounting Office, "Campaign Finance Reform: Early Experiences of Two States that Offer Full Public Funding for Political Candidates," May 2003.

[52] Whitt, W. (1980), "Representation and Approximation of Noncooperative Sequential Games," *SIAM Journal of Control and Optimization* 18(1), 33–48.

8 Online Appendix [For Online Publication]

In the Online Appendix, we provide the proofs that we ommitted from the main text as well as details regarding the model, estimation, data construction and applications to other environments. In Section 8.1, we describe the model of open-seat elections. In Section 8.2, we prove Proposition 2-2 (Sufficient statistics). In Section 8.3, we prove Proposition 1 (Injectivity). In Section 8.4, we discuss how we forward-simulate the continuation value. Section 8.5 explains data construction. In Section 8.6, we provide further details of the estimation procedure. In Section 8.7, we show that our approach extends to an environment in which q_I is time-varying, and one in which we observe few uncontested elections. In Section 8.8, we show that at the estimated parameter values, the challenger's value function, v_C, is not too decreasing in q_C in our data: a condition that ensures that the challenger's entry decision follows the cutoff strategy.

8.1 Description of the Model of Open-Seat Elections

In an open-seat election, challengers from both parties decide whether or not to enter. Consequently, the process that determines the valence of the challengers (Stages 1 and 2(a) in Figure 1) applies to both parties. The value function of candidate i once she becomes the party nominee is as follows:

$$v_O(X, q_i, q_j) = \max_{w_i' \geq 0, d_i \geq 0} B \cdot \Pr(vote_i > 0.5) - C_O(w_i' + d_i, q_i)$$
$$+ H_O(d_i) + \delta \Pr(vote_i > 0.5) \mathbb{E}_{\mathbf{s}' | w_i', ten_i = 0, X}[V_I(\mathbf{s}')],$$

where

$$vote_i = \beta_O(\ln d_i - \ln d_j) + \beta_{pt}(pt \times D_i) + \beta_{ue}(ue \times D_i \times D_P) + q_i - q_j + \varepsilon, (i \neq j).$$

$C_O(\cdot)$ and $H_O(\cdot)$ are the cost of fund-raising and the benefit of spending in open-seat elections.[35] While the formulation is analogous to that of elections with incumbents, the coefficient of spending on the vote share is restricted to be the same across the two candidates and w_I and ten_I are not included in the state. If candidate i wins, she becomes the incumbent next period and the new state is $\mathrm{s}' = \{q_I = q_i, w_I = 1.1 w_i', ten_I = 2, X'\}$.

[35] In our empirical specification, we assume that $C_O(\cdot) = C_C(\cdot)$ and $H_O(\cdot) = H_C(\cdot)$.

Open-seat candidates make entry decisions by comparing their expected return from entry and the cost of entry, κ_O. The ex-ante value function can be expressed as follows:

$$V_O(X, q_i) = \max\left\{ p_O(q_i, X) \int v_O(X, q_i, q_j) dG_{q_j}(q_j|X) - \kappa_O, 0 \right\}, (i \neq j),$$

where $p_O(q_i, X)$ denotes the ex-ante probability of a challenger with valence q_i being selected as a party nominee, defined analogously to Expression (7). As candidates do not know the valence of the candidate from the opponent party, we take expectation over v_O with respect to G_{q_j}, which is the valence distribution of the opponent in the general election.

8.2 Sufficient Statistics when N Depends on s

We give a proof of Proposition 2-2 below.

Proposition 2-2 (Sufficient statistic): *Suppose that N is distributed according to a CDF $F_N(\cdot|\mathbf{s})$ that is fully characterized by the first L moments, $\mathbb{E}[N|\mathbf{s}], \cdots, \mathbb{E}[N^L|\mathbf{s}]$. If $\mathbb{E}[N^{L+1}|\mathbf{s}]$ is weakly increasing in the first L moments, then $L+1$ moments of M, $m_M = (\mathbb{E}[M|\mathbf{s}], \cdots, \mathbb{E}[M^{L+1}|\mathbf{s}])$ are sufficient statistics for $G_{q_C}(\cdot|\mathbf{s})$. Generically, $L+2$ moments of M, $\tilde{m}_M = (\mathbb{E}[M|\mathbf{s}], \cdots, \mathbb{E}[M^{L+2}|\mathbf{s}])$ are sufficient statistics for $G_{q_C}(\cdot|\mathbf{s})$.*

Recall from expression (9) that $G_{q_C}(q_C|\mathbf{s})$ has the following expression:

$$G_{q_C}(t|\mathbf{s}) = \mathbb{E}_N \left[\sum_{M=1}^{N} Bin(N, M; 1 - F_{q_C}(\bar{q}_C)) \frac{\int_{\bar{q}_C}^{t} \int_{\bar{q}_C}^{+\infty} \cdots \int_{\bar{q}_C}^{+\infty} M\pi(q_{C,n}, \mathbf{q}_{C,-n})(dF_{q_C})^M}{(1 - F_{q_C}(\bar{q}_C))^M} \middle| \mathbf{s} \right].$$

By assumption, the distribution of N is characterized by the first L moments, $\mathbb{E}[N|\mathbf{s}], \cdots, \mathbb{E}[N^L|\mathbf{s}]$. Hence, $\bar{q}_C(\mathbf{s})$ and $\mathbb{E}[N|\mathbf{s}], \cdots, \mathbb{E}[N^L|\mathbf{s}]$ are sufficient statistics. We now want to show that we can take $\mathbb{E}[M|\mathbf{s}], \cdots, \mathbb{E}[M^{L+2}|\mathbf{s}]$ as sufficient statistics.

Because each potential entrant decides whether or not to enter independently of other potential entrants, the expected number of actual entrants in the Primary, $\mathbb{E}[M|\mathbf{s}]$, can be written as the product of the expected number of potential entrants $\mathbb{E}[N|\mathbf{s}]$ and the probability of entry, $1 - F_{q_C}(\bar{q}_C(\mathbf{s}))$:

$$\mathbb{E}[M|\mathbf{s}] = \mathbb{E}[N|\mathbf{s}] \times (1 - F_{q_C}(\bar{q}_C(\mathbf{s}))).$$

Using the fact that the moment generating function of M given N takes the following form, $\mathbb{E}[e^{tM}|N] = (1 - p + pe^t)^N$,

$$\mathbb{E}[M(M-1)|\mathrm{s}] = \mathbb{E}[N(N-1)|\mathrm{s}] \times (1 - F_{q_C}(\bar{q}_C(\mathrm{s})))^2$$
$$\vdots$$
$$\mathbb{E}[M(M-1)\cdots(M-L-1)|\mathrm{s}] = \mathbb{E}[N(N-1)\cdots(N-L-1)|\mathrm{s}] \times (1 - F_{q_C}(\bar{q}_C(\mathrm{s})))^{L+2}.$$

The above expressions are equations in $(\mathbb{E}[N^k|\mathrm{s}])_{k=1}^{L+2}$, $(\mathbb{E}[M^k|\mathrm{s}])_{k=1}^{L+2}$ and $F_{q_C}(\bar{q}_C(\mathrm{s}))$. If the distribution of N is fully characterized by the first L moments, then $\mathbb{E}[N^{L+1}|\mathrm{s}]$ and $\mathbb{E}[N^{L+2}|\mathrm{s}]$ are deterministic functions of $\mathbb{E}[N|\mathrm{s}]$,$\cdots$, $\mathbb{E}[N^L|\mathrm{s}]$. Hence, if we take $(\mathbb{E}[M^k|\mathrm{s}])_{k=1}^{L+2}$ as known, the above expressions can be thought of as $(L+2)$ equations in $(\mathbb{E}[N^k|\mathrm{s}])_{k=1}^{L}$, and $F_{q_C}(\bar{q}_C(\mathrm{s}))$. Because there are more equations $(L+2)$ than unknowns $(L+1)$, there is a unique profile $(\mathbb{E}[N^k|\mathrm{s}])_{k=1}^{L+2}$, and $F_{q_C}(\bar{q}_C(\mathrm{s}))$ that satisfies the above expressions, which implies that $(\mathbb{E}[M^k|\mathrm{s}])_{k=1}^{L+2}$ are sufficient statistics.

We now discuss the special case in which the distribution of N is fully characterized by its mean. We can rearrange the expressions corresponding to $\mathbb{E}[M|\mathrm{s}]$ and $\mathbb{E}[M(M-1)|\mathrm{s}]$ to obtain the following expressions,

$$\mathbb{E}[N|\mathrm{s}] = \mathbb{E}[M|\mathrm{s}]/p(\mathrm{s})$$
$$\mathbb{E}[M^2|\mathrm{s}] = \mathbb{E}[N^2|\mathrm{s}]p(\mathrm{s})^2 + (1 - p(\mathrm{s}))\mathbb{E}[M|\mathrm{s}]$$

E[M(M-1)—s] =

the first where the dependence of actual entrants in the Primary. $G_{q_C}(t|\mathrm{s})$ on $r(\mathrm{s})$ is implicit through the expectation over N. Note that $\bar{q}_C(\mathrm{s})$ and $r(\mathrm{s})$ are sufficient statistics, by inspection. That is, as long as $\bar{q}_C(\mathrm{s}) = \bar{q}_C(\mathrm{s}')$ and $r(\mathrm{s}) = r(\mathrm{s}')$, we have $G_{q_C}(t|\mathrm{s}) = G_{q_C}(t|\mathrm{s}')$. In order to show that $P_e(\mathrm{s})$ and $\mathbb{E}[M|\mathrm{s}]$ are also sufficient statistics for $G_{q_C}(t|\mathrm{s})$, it then suffices to show that whenever $P_e(\mathrm{s}) = P_e(\mathrm{s}')$ and $\mathbb{E}[M|\mathrm{s}] = \mathbb{E}[M|\mathrm{s}']$, we have $\bar{q}_C(\mathrm{s}) = \bar{q}_C(\mathrm{s}')$ and $r(\mathrm{s}) = r(\mathrm{s}')$.

Lemma 1 *Suppose that $F_N(\cdot|\mathrm{s})$ is given by the negative binomial distribution, $F_N(\cdot|\mathrm{s}) =$ NB$(r(\mathrm{s}), p)$, where $r(\cdot)$ can be an arbitrary function of* s. *Then, $P_e(\mathrm{s})$ and $\mathbb{E}[M|\mathrm{s}]$ are sufficient statistics for $G_{q_C}(q_C|\mathrm{s})$.*

Proof. Recall from expression (9) that $G_{q_C}(q_C|\mathbf{s})$ has the following expression:

$$
\begin{aligned}
&G_{q_C}(t|\mathbf{s}) \\
&\quad = \mathbb{E}_N\left[\sum_{M=1}^{N} Bin(N,M;1-F_{q_C}(\bar{q}_C))\frac{\int_{\bar{q}_C}^{t}\int_{\bar{q}_C}^{+\infty}\cdots\int_{\bar{q}_C}^{+\infty} M\pi(q_{C,n},\mathbf{q}_{C,-n})(dF_{q_C})^M}{(1-F_{q_C}(\bar{q}_C))^M}\right|\mathbf{s}\Bigg],
\end{aligned}
$$

where the dependence of $G_{q_C}(t|\mathbf{s})$ on $r(\mathbf{s})$ is implicit through the expectation over N. Note that $\bar{q}_C(\mathbf{s})$ and $r(\mathbf{s})$ are sufficient statistics, by inspection. That is, as long as $\bar{q}_C(\mathbf{s}) = \bar{q}_C(\mathbf{s}')$ and $r(\mathbf{s}) = r(\mathbf{s}')$, we have $G_{q_C}(t|\mathbf{s}) = G_{q_C}(t|\mathbf{s}')$. In order to show that $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ are also sufficient statistics for $G_{q_C}(t|\mathbf{s})$, it then suffices to show that whenever $P_e(\mathbf{s}) = P_e(\mathbf{s}')$ and $\mathbb{E}[M|\mathbf{s}] = \mathbb{E}[M|\mathbf{s}']$, we have $\bar{q}_C(\mathbf{s}) = \bar{q}_C(\mathbf{s}')$ and $r(\mathbf{s}) = r(\mathbf{s}')$.

Recall from expression (**??**) that

$$
\begin{aligned}
\mathbb{E}[M|\mathbf{s}] &= \mathbb{E}_N\left[N(1-F_{q_C}(\bar{q}_C))|\,\mathbf{s}\right] \\
&= (1-F_{q_C}(\bar{q}_C(\mathbf{s})))\mathbb{E}_N\left[N|\mathbf{s}\right].
\end{aligned}
$$

When $F_N(\cdot|\mathbf{s})$ is the negative binomial distribution, the expression for $\mathbb{E}[M|\mathbf{s}]$ becomes

$$
\mathbb{E}[M|\mathbf{s}] = (1-F_{q_C}(\bar{q}_C(\mathbf{s})))\frac{pr(\mathbf{s})}{1-p}, \tag{15}
$$

where we use the fact that the mean of the negative binomial distribution NB(r,p) is $\frac{pr}{1-p}$.

On the other hand, $P_e(\mathbf{s})$ has the following form (see expression (**??**)):

$$
\begin{aligned}
P_e(\mathbf{s}) &= \mathbb{E}_N\left[1-F_{q_C}(\bar{q}_C(\mathbf{s}))^N\big|\,\mathbf{s}\right] \\
&= 1-\sum_{N=0}^{+\infty}\begin{pmatrix} N+r(\mathbf{s})-1 \\ N \end{pmatrix} p^N(1-p)^{r(\mathbf{s})}F_{q_C}(\bar{q}_C(\mathbf{s}))^N \\
&= 1-\sum_{N=0}^{+\infty}\begin{pmatrix} N+r(\mathbf{s})-1 \\ N \end{pmatrix} (F_{q_C}(\bar{q}_C(\mathbf{s}))p)^N(1-F_{q_C}(\bar{q}_C(\mathbf{s}))p)^{r(\mathbf{s})}\times\frac{(1-p)^{r(\mathbf{s})}}{(1-F_{q_C}(\bar{q}_C(\mathbf{s}))p)^{r(\mathbf{s})}} \\
&= 1-\left(\frac{1-p}{1-F_{q_C}(\bar{q}_C(\mathbf{s}))p}\right)^{r(\mathbf{s})},
\end{aligned} \tag{16}
$$

where the second line follows from the definition of the probability mass function of the negative binomial distribution, and the fourth line follows from the fact that the probability mass function sums up to one. In order to show that $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ are sufficient statistics, it suffices to show that we can uniquely solve for $r(\mathbf{s})$ and $\bar{q}_C(\mathbf{s})$ in equations (15) and (16) as functions of $\mathbb{E}[M|\mathbf{s}]$ and $P_e(\mathbf{s})$.[36]

With that in mind, we first take expression (15) and solve for $F_{q_C}(\bar{q}_C(\mathbf{s}))$:

$$F_{q_C}(\bar{q}_C(\mathbf{s})) = 1 - \frac{\mathbb{E}[M|\mathbf{s}](1-p)}{pr(\mathbf{s})}. \tag{17}$$

We then substitute out $F_{q_C}(\bar{q}_C(\mathbf{s}))$ from expression (16):

$$\begin{aligned} P_e(\mathbf{s}) &= 1 - \left(\frac{1-p}{1 - \left(1 - \frac{\mathbb{E}[M|\mathbf{s}](1-p)}{pr(\mathbf{s})}\right) p} \right)^{r(\mathbf{s})} \\ &= 1 - \left(\frac{1-p}{1 - p + \frac{\mathbb{E}[M|\mathbf{s}](1-p)}{r(\mathbf{s})}} \right)^{r(\mathbf{s})} = 1 - \left(\frac{1}{1 + \frac{\mathbb{E}[M|\mathbf{s}]}{r(\mathbf{s})}} \right)^{r(\mathbf{s})}. \end{aligned} \tag{18}$$

If we can show that the right-hand side of expression (18) is monotone in $r(\mathbf{s})$, this implies that we can express $r(\mathbf{s})$ uniquely as a function of $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$. The proof would then be done because, together with equation (17), the monotonicity would ensure that both $r(\mathbf{s})$ and $\bar{q}_C(\mathbf{s})$ are expressed uniquely as a function of $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$.

In order to show that the right-hand side of expression (18) is monotone in $r(\mathbf{s})$, consider a function $f(x)$ defined as follows:

$$f(x) = \frac{1}{x}\ln(1+x), \; (x > 0)$$

It is easy to see from Figure 8 that $f(x)$ is monotone decreasing in x for any $x > 0$.

Given that $f(x)$ is monotone decreasing, $\exp(-\alpha f(\alpha/x))$ is monotone decreasing for $x > 0$ for any constant $\alpha > 0$, where

$$\exp(-\alpha f(\alpha/x)) = \left(\frac{1}{1+\frac{\alpha}{x}}\right)^x.$$

By inspection, the right-hand side of expression (18) is monotone increasing in $r(\mathbf{s})$, and we are done. ■

Note that even if we do not impose any functional form assumption on $F_N(\cdot|\mathbf{s})$, $\bar{q}_C(\mathbf{s})$

[36]Suppose that we can uniquely solve for $r(\mathbf{s})$ and $\bar{q}_C(\mathbf{s})$ as functions of $\mathbb{E}[M|\mathbf{s}]$ and $P_e(\mathbf{s})$. Then, if we have $\mathbb{E}[M|\mathbf{s}] = \mathbb{E}[M|\mathbf{s}']$ and $P_e(\mathbf{s}) = P_e(\mathbf{s}')$, we would have $r(\mathbf{s}) = r(\mathbf{s}')$ and $\bar{q}_C(\mathbf{s}) = \bar{q}_C(\mathbf{s}')$. Given that $r(\mathbf{s})$ and $\bar{q}_C(\mathbf{s})$ are sufficient statistics, this means that $\mathbb{E}[M|\mathbf{s}]$ and $P_e(\mathbf{s})$ are also sufficient statistics.

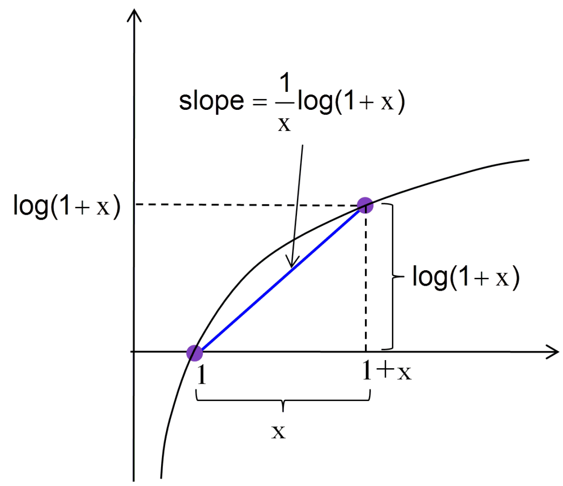

Figure 8: A graphical explanation of why $f(x)$ is monotone. Note that $\frac{1}{x}\ln(1+x)$ corresponds to the slope of $\ln(t)$ between $t = 1$ and $t = 1 + x$. Because $\ln(t)$ is concave, $f(x)$ is decreasing.

and $F_N(\cdot|\mathrm{s})$ are still sufficient statistics for $G_{q_C}(t|\mathrm{s})$. Hence, if we relax our distributional assumption on $F_N(\cdot|\mathrm{s})$ so that $F_N(\cdot|\mathrm{s})$ now depends on $r_1(\mathrm{s})$,...,$r_L(\mathrm{s})$ – i.e., $F_N(\cdot|\mathrm{s}) = F_N(\cdot|r_1(\mathrm{s}),...,r_L(\mathrm{s}))$ – then $\bar{q}_C(\mathrm{s})$, $r_1(\mathrm{s})$,...,$r_L(\mathrm{s})$ would be sufficient statistics. It is intuitive to see that we can obtain a similar sufficient statistic result following an argument analogous to the one above. When $L > 1$, we would need more functionals of $F_N(\cdot|\mathrm{s})$ for the sufficient statistic result to hold: for example, we may use quantiles of M to condition on in addition to $P_e(\mathrm{s})$ and $\mathbb{E}[M|\mathrm{s}]$.

8.3 Inversion of q_I from Uncontested Periods

In this Section, we give a proof of Proposition 1.

Proposition 1 (Injectivity): *Assume that the marginal cost of raising money, $\frac{\partial}{\partial x}\tilde{C}_I(x, q_I)$, is strictly decreasing with respect to q_I. Then, the policy functions of an uncontested incumbent, $\{d_I(\mathrm{s}),w'_I(\mathrm{s})\}$, are one-to-one from q_I to (d_I, w'_I), holding other state variables fixed.*

Proof. Note that the first-order condition for d_I implies

$$\underbrace{\frac{\partial}{\partial d_I}\tilde{H}_I(d_I)}_{\text{MB of spending}} - \underbrace{\frac{\partial}{\partial d_I}\tilde{C}_I(w_I' + d_I - w_I, q_I)}_{\text{MC of fund-raising}} = 0.$$

Now suppose to the contrary that the mapping from q_I to (d_I, w_I') is not one-to-one, so that q_I and $\tilde{q}_I$ $(q_I > \tilde{q}_I)$ both map to (d_I, w_I'). Then,

$$\frac{\partial}{\partial d_I}\tilde{H}_I(d_I) = \frac{\partial}{\partial d_I}\tilde{C}_I(w_I' + d_I - w_I, q_I)$$

and

$$\frac{\partial}{\partial d_I}\tilde{H}(d_I) = \frac{\partial}{\partial d_I}\tilde{C}_I(w_I' + d_I - w_I, \tilde{q}_I)$$

However, given that $\frac{\partial}{\partial d_I}\tilde{C}_I(\cdot,\cdot)$ is strictly decreasing in the second argument,

$$\frac{\partial}{\partial d_I}\tilde{C}_I(w_I' + d_I - w_I, q_I) < \frac{\partial}{\partial d_I}\tilde{C}_I(w_I' + d_I - w_I, \tilde{q}_I),$$

which is a contradiction. ■

Proposition 1 allows us to invert the policy function of uncontested incumbents to express the unobserved incumbent valence, q_I, as a function of the state and incumbent's actions in uncontested periods, $\bar{\mathbf{s}}_U$. In our empirical analysis, we also use the following lemma that the specific functional form we use for $\tilde{C}_I$ and $\tilde{H}_I$ allows us to simplify the mapping $q_I(\bar{\mathbf{s}}_U)$.

Lemma 2 *Suppose that the mapping from q_I to (d_I, w_I') is one-to-one given other state variables according to Proposition 1. If we further assume that $\tilde{C}_I(y; q_I) = c(q_I)(\ln y)^2$, where $c(\cdot)$ is a decreasing function and $\tilde{H}_I(y) = \gamma_U\sqrt{\ln y}$ as specified in our estimation, the inverse mapping from (d_I, w_I') to q_I simplifies to*

$$q_I = c^{-1}\left(\frac{\gamma_U}{4}\frac{w_I' + d_I - w_I}{(\ln d_I)^{1/2} d_I \ln(w_I' + d_I - w_I)}\right).$$

Proof. Suppose that $\tilde{C}_I(y; q_I) = c(q_I)(\ln y)^2$, and $\tilde{H}_I(y) = \gamma_U\sqrt{\ln y}$. Substituting these expressions into the first-order condition, we obtain

$$\begin{aligned}&\frac{\gamma_U}{2}(\ln d_I)^{-1/2}(d_I)^{-1} - 2c(q_I)(\ln fr_I)(fr_I)^{-1} = 0 \\ \Longleftrightarrow\ & c(q_I) = \frac{\gamma_U}{4}(\ln d_I)^{-1/2}(d_I)^{-1}(\ln fr_I)^{-1} fr_I \\ \Longleftrightarrow\ & q_I = c^{-1}\left(\frac{\gamma_U}{4}\frac{fr_I}{(\ln d_I)^{1/2} d_I \ln fr_I}\right),\end{aligned} \tag{19}$$

where fr_I denotes the amount raised ($fr_I = w'_I + d_I - w_I$). We use the fact that $c(\cdot)$ is monotone to obtain the last line of the expression. Expression (19) implies that if spending d_I and amount raised are the same (given other state variables), q_I has to be the same. This concludes the proof. ■

The fact that we can control for q_I just by conditioning on a one-dimensional object, $z_U \equiv \frac{fr_I}{(\ln d_I)^{1/2} d_I \ln fr_I}$, simplifies our estimation immensely. It would be very difficult to implement our procedure if we had to condition on the full vector of actions and state variables, $\bar{\mathbf{s}}_U$.

8.4 Forward-Simulation of the Continuation Value

In this section, we discuss how we forward-simulate the continuation value. As we discussed in Section 3.2, our idea is based on Hotz, Miller, Sanders, and Smith (1994) and Bajari, Benkard and Levin (2007). These papers propose a method of simulating the value function by first estimating the policy function and then using the policy function to generate sample paths of outcomes and actions, which can then be averaged to compute the continuation value. Because we do not observe q_C in contested periods, we estimate the distribution of the actions and outcomes conditional on observed state variables instead of the actual policy function. Below, we describe the details of our procedure.

8.4.1 Estimation of the transition probability of the states

We assume an exogenous AR(1) process for $\{ue, pt\}$ as $X^k_{t+1} = \alpha^k_0 + \alpha^k_1 X^k_t + \xi^k_{t+1}$, and $\xi^k_{t+1} \sim N(0, \sigma^k_\xi)$, where $X^k \in \{ue, pt\}$.[37] Regarding the evolution of D_P, we assume that (1) D_P remains the same next period with probability 0.75 in a general election when the president is running for the second term; (2) D_P remains the same with probability 0.5

[37] We estimated $\alpha^{ue}_0 = 0.02$, $\alpha^{ue}_1 = 0.70$, $\sigma^{ue}_\xi = 0.0002$ and $\alpha^{pt}_0 = 0.01$, $\alpha^{pt}_1 = 0.85$, $\sigma^{pt}_\xi = 0.07$.

when the incumbent president is in his second term; and (3) D_P remains the same next period with probability one if the election is a Midterm election.

8.4.2 Estimation of the distribution of actions conditional on observed state variables

The second set of objects we estimate are the projections of the policy functions on observed state variables. The relevant objects we estimate are as follows:

Distribution of d_I and fr_I conditional on s in contested periods Recall that the equilibrium spending and amount raised by the incumbent in contested periods maps (s, q_C) to a non-negative number, where $\mathrm{s} \equiv \{q_I(\bar{\mathrm{s}}_U), w_I, ten_I, X\}$.[38] The projection of the policy function on s is just the conditional distribution of d_I and fr_I in contested periods given observable states, denoted as $F_{d_I}(\cdot|\mathrm{s})$ and $F_{fr_I}(\cdot|\mathrm{s})$, respectively. We use a (first-order) Hermite series approximation to estimate the conditional distribution, by nonparametric maximum likelihood (Gallant and Nychka 1987). Because ten_I is a discrete variable, we estimate separate distributions for $ten_I \in [1, 3]$, $ten_I \in [4, 7]$, and $ten_I \in [7, \infty]$.

Distribution of w_I' conditional on s and $vote_I > 0.5$ in contested periods We estimate the distribution of incumbent savings in contested periods in the same way as spending and fund-raising. However, in order to simulate the value function, we need the distribution of savings *conditional on winning*. Hence we estimate $F_{w_I'}(\cdot|\mathrm{s}, \{vote_I > 0.5\})$, where $\{vote_I > 0.5\}$ corresponds to the event that the vote share of the incumbent is above 50 percent.[39]

Policy functions in uncontested periods We approximate the amount of spending and saving in uncontested periods by least squares. The regressors include a constant, X, ten_I and B-spline of z_U. We also include quadratic terms as well as interactions of these variables. In uncontested periods, we can estimate the policy function itself because the state variables are all observed.

[38]In practice, we use lemma 2 in Appendix 8.3 and use a scalar variable z_U in place of $\bar{\mathrm{s}}_U$.

[39]In practice, instead of directly estimating incumbent's savings conditional on winning, we estimate incumbent's spending and fund-raising conditional on winning and derive the distribution of saving using incumbent's budget constraint. This helps to ensure that the estimated distribution of savings is internally consistent with that of spending and fund-raising.

8.4.3 Estimation of the distribution of outcomes conditional on observed state variables

Lastly, we estimate the retirement probability, $\lambda(\mathbf{s})$, and the probability that the incumbent wins, $P_{win}(\mathbf{s})$.

Retirement probability, $\lambda(\mathbf{s})$ We estimate the probability that the incumbent retires as a function of s. We specify $\lambda(\mathbf{s})$ to be a nonparametric function of ten_I.[40]

Probability of winning, $P_{win}(\mathbf{s})$, in contested periods We also estimate the probability that the incumbent wins in contested periods given s, denoted as $P_{win}(\mathbf{s})$. $P_{win}(\mathbf{s})$ is estimated by a Probit, with the regressors being $\ln w_I$, $(\ln w_I)^2$, $\ln ten_I$, $ue \times D_I \times D_P$, $pt \times D_I$ and B-spline bases of z_U.

8.4.4 Computation of the Continuation Value

Once we obtain estimates of the distribution of actions and outcomes conditional on observed states, it is possible to simulate the continuation value for each profile of parameters. The key to our approach is that the incumbent's utility does not depend directly on q_C, which is unobservable, but only indirectly through actions and outcomes such as d_I, fr_I, etc. We compute the continuation value, $\mathbb{E}[V_I(\mathbf{s}')]$ starting from a given s as follows:

1. Randomly draw X', given X using the estimated transition matrix, which gives us a new state vector, $\mathbf{s}' = \{q_I, w_I, ten_I, X'\}$. Draw a random variable U_{RET} from a uniform $\mathrm{U}(0,1)$. If U_{RET} is less than $\lambda(\mathbf{s}')$, then the incumbent retires and we terminate the process.

2. Draw a random variable U_{ENT} from a $\mathrm{U}(0,1)$. If U_{ENT} is less than the probability of entry, i.e., $U_{ENT} \leq P_e(\mathbf{s}')$, then there is entry (Recall that P_e is estimated in Sec 3.1.2). If $U_{ENT} > P_e(\mathbf{s}')$, then there is no entry.

3. Depending on whether or not there is challenger entry in the previous step, draw d_I and fr_I using the conditional distributions ($F_{d_I}(\cdot|\mathbf{s}')$ and $F_{fr_I}(\cdot|\mathbf{s}')$) estimated above or compute them using the estimated policy function in uncontested periods. In case of entry, further draw a random variable U_{win} from a $\mathrm{U}(0,1)$.

[40] We assume that $\lambda(ten_I)$ is constant for all $ten_I \geq 10$.

4. The period utility function is computed as $\tilde{u}_I = B - \tilde{C}_I(fr_I, q_I) + \tilde{H}_I(d_I)$ in the case of no entry. If there is entry, the period utility is either $u_I = B - C_I(fr_I, q_I) + H_I(d_I)$ or $u_I = -C_I(fr_I, q_I) + H_I(d_I)$, depending on whether U_{win} is smaller or bigger than $P_{win}(\mathbf{s}')$. A draw of U_{win} smaller than $P_{win}(\mathbf{s}')$ is interpreted as a victory for the incumbent, and a larger value is a loss of the incumbent.

5. Terminate the process if the incumbent loses to the entrant. Otherwise, draw w'_I from $F_{w'_I}(\cdot|\mathbf{s}', \{v_I > 0.5\})$. This determines the amount of savings.

6. The state variables become $\{q_I, w'_I, ten_I + 1, X'\}$. Go back to step 1 and repeat until termination. Take the discounted sum of u_I.

7. Repeat steps 1 through 6 and take the average.

Note that for computing the continuation value, knowledge of the marginal distributions of the actions is enough, and not the joint distribution. This follows from the additive separability of u_I and it greatly simplifies the computation.

8.4.5 Computation of the Derivatives of Continuation Value and Challengers' Continuation Value

In evaluating the right hand side of expression (12), we need to compute the derivative of the value function with respect to w_I. To do so, we approximate the value function with polynomials of the state variables and use its derivative with respect to w_I.[41] We also use this polynomial to evaluate challengers' continuation payoff. This is possible because the challenger becomes the incumbent conditional on winning.

8.5 Data Construction

We constructed the sample we use for our estimation as follows: We first drop all House elections in Louisiana.[42] We also drop elections in Texas in 1996 which were deemed unconstitutional in the Supreme Court.[43] We also drop special elections held outside of the

[41] The alternative approach is to use numerical differentiation, but we found the numerical derivative to be less stable, depending heavily on the step size. This may be because we are not allowing the distribution of actions to be sufficiently flexible in our estimation.

[42] Louisiana has a run-off election unlike any other U.S. state.

[43] The Congressional Elections that were affected by the Supreme Court ruling are TX03, TX05, TX06, TX07, TX08, TX09, TX18, TX22, TX24, TX25, TX26, TX29 and TX30 in 1996.

regular election cycle, elections that occur right after special elections, instances in which two incumbents run against each other, and elections in which a major scandal broke out.[44] Some observations were also dropped because of missing data.[45] We also drop elections in which either candidate spends less than $\$5,000$ and saves less than $\$5,000$ since these elections are not competitive and resemble uncontested elections. Lastly, we drop elections in which either candidate incumbent spends or saves more than $\$1.2$ million since unusually large campaign budget is invariably for running for higher offices. As a consequence of these conditions, we drop 800 elections. We are left with a base sample of 2,459 contested elections, 580 uncontested elections and 356 open-seat elections.[46]

Creation of Partisanship Measure One of the variables of the vote share equation is the partisanship measure of the District, pt. In order to construct this variable, we follow Levendusky et al (2008) and regress the log difference in the district-level vote shares of the Democrats and the Republicans in the Presidential election between 1952 and 2008 on the following variables; fraction of the population who are 65 or above, fraction of blue-collar workers, fraction of foreign-born people, the median income, population density, unemployment rate, fraction of Blacks and Hispanics and its interactions with a dummy variable that corresponds to the South. All of the regressors are in the log. In addition, we include a dummy variable that corresponds to whether or not the candidate is from the state in which the District is located, and year and state fixed effects. The partisanship measure is obtained as the fitted value of the regression.

[44]The list of elections that were dropped because of a scandal are CA17 (1990), MA04 (1990), MN06 (1992), NY15 (1992) and NY15 (2000). These events were identified by going through the biography of candidates in the CQ press Congressional Collection.

[45]Some of the entries in the FEC data set is clearly incorrect. Some candidates are listed as having run in a wrong State, for example. Most of these missing data is easily identifiable because the vote shares do not add up to one or there are multiple candidates from the same party. Where the accuracy of the data is suspect, Open Secrets (http://www.opensecrets.org/) was used as a cross-check in order to correct the mistakes. The full list of changes that were made is available upon request.

[46]At a certain stage of the estimation procedure, we may use only a subset of the sample. For example, to estimate the candidates' utility functions such as $H_I(\cdot)$, the identification requires that we find two values of K, each derived from the incumbent's first-order conditions for saving and spending (see Section 3.2), indicating that both first-order conditions need to be satisfied with equality. Thus, we only use incumbents whose spending and savings are both strictly positive in this stage.

8.6 Details on the Estimation

We now discuss the details on the estimation that we omit from the main text. The estimation follows a multi-step procedure ordered as follows.

Estimation of $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ We estimate $P_e(\mathbf{s})$ with a Probit, and $\mathbb{E}[M|\mathbf{s}]$ by a linear regression. We specify both $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$ as a function of a constant, $\ln w_I$, $(\ln w_I)^2$, $ue \times D_I \times D_P$, $pt \times D_I$, $\ln ten_I$, $1\{\text{President 1st term}\}$, $1\{\text{Midterm}\}$ and $1\{\text{President 1st term}\} \times 1\{\text{Midterm}\}$, where $1\{\text{President 1st term}\}$ and $1\{\text{Midterm}\}$ are dummy variables for the event that the current president is in his first term of office, and for the event that the election is a Midterm election, respectively. We also include B-spline bases of $z_U = \frac{fr_I}{(\ln d_I)^{1/2} d_I \ln fr_I}$ and their interaction terms with $\ln w_I$. We take 7 knots, corresponding to (1/8,...,7/8) quantiles of z_U.

Estimation of the Vote Share Equation We approximate $g(P_e(\mathbf{s}), \mathbb{E}[M|\mathbf{s}])$ with a second-order polynomial of $P_e(\mathbf{s})$ and $\mathbb{E}[M|\mathbf{s}]$. We also approximate $q_I(z_U)$ as a polynomial of order four in z_U. We then project the residual of the vote share equation on a set of basis functions consisting of pre-determined variables, which include all variables in the vote share equation except for $\ln d_I$ and $\ln d_C$, $\ln w_I$, $(\ln w_I)^2$, $ue^2 \times D_I \times D_P$, pt, $(\ln ten_I)^2$, $1\{\text{President 1st term}\}$, $1\{\text{Midterm}\}$, $1\{\text{President 1st term}\} \times 1\{\text{Midterm}\}$ and B-spline bases of z_U. We then minimize the squared sum.

Estimation of Components of Candidates' Payoffs and σ_ε^2 We estimate the components of the candidates' payoff function and σ_ε^2 using moments constructed from the first-order conditions and orthogonality conditions implied by the model. For each parameter value, we first simulate the continuation value of the incumbents, $\mathbb{E}_{X'|X}[V_I(\mathbf{s}')]$, and compute its derivative, $\frac{\partial}{\partial w_I'} \mathbb{E}_{X'|X}[V_I(\mathbf{s}')]$, according to the method described in Appendix 8.4. We then invert the incumbent's first-order condition regarding saving (expression (12)) to obtain the value of K, and expression (13) to obtain the value of q_C. Finally, we substitute out K and q_C in expressions (11) and the two first-order conditions of the challengers. The three first-order conditions are then only a function of observed actions, observed states (note that q_I has been estimated in the previous step), and model parameters. We stack those three first-order conditions with the moment conditions corresponding to the incumbents' first-order conditions in uncontested periods and 14 extra orthogonality conditions.[47][48] We use identity matrix to weight between the first-order conditions.

For some of the observations, we encounter trouble inverting Φ in expression (12) to obtain K, because the argument inside Φ^{-1} exceeds 1. This corresponds to the case in which the implied winning probability of the incumbent exceeds 1. Consequently, we replace the value of $\Phi^{-1}(\cdot)$ with $1 - 10^{-6}$ when the argument is above 1. At the estimated parameters, the argument inside Φ^{-1} is bigger than 1 in 281 elections out of 408 total elections. We acknowledge that this is not ideal. However, note that even at the true parameter values, the argument of Φ^{-1} can exceed 1 when the other parameters (such as the distribution of outcomes and actions) are estimated with noise. Given that there are many elections in which the incumbent is almost sure to win, even small estimation errors make the term inside Φ^{-1} exceed 1.

Estimation of Parameters in Open-seat Elections We estimate β_O and q_O by following the procedure identical to the case of elections with incumbents, except that we only use the three first-order conditions as moments. The value functions are calculated by the polynomial approximation of the incumbents' value function obtained above.

Estimation of Candidate Valence in All Elections Once all the model parameters are estimated, we recover q_I, q_C and q_O for all candidates in our sample. We run a GMM similar to the one we used to estimate payoffs, but now payoffs are known and the parameters to be estimated are the valence measures of each candidate. We use as moment conditions first-order conditions of both candidates from each contested and open-seat election, as well as that of the incumbent from each uncontested election.

When we recover the valence using the first-order conditions, there is some degree of freedom in terms of which first-order conditions we use. This is because there are more equations than unknowns per election, and because we restrict the valence measure of each

[47]To utilize information available in the vote share equation, we include orthogonality conditions between ε and each of the 8 right hand side variables in the vote share equations as moments. We also include 5 moment conditions corresponding to the orthogonality between $q_C - g(P_e(\mathbf{s}), \mathbb{E}[M|\mathbf{s}])$ and predetermined states ($ue \times D_I \times D_P$, $pt \times D_I$, $\ln ten_I$, q_I and a constant). Note that $g(P_e(\mathbf{s}), \mathbb{E}[M|\mathbf{s}])$ is the conditional expectation of q_C estimated in the first stage, and that the observed states must be orthogonal to the residual in q_C after controlling for its conditional expectation. Finally, we include the restriction that the variance of the error term obtained in the estimation of the vote share equation equals the sum of σ_ε^2 and the variance of $q_C - g(P_e(\mathbf{s}), \mathbb{E}[M|\mathbf{s}])$.

[48]In order to maintain stability of estimation, we also impose the restriction that the continuation payoffs and its derivative with respect to saving are nonnegative.

candidate to be constant across elections. In practice, we minimize the sum of squared deviation of all the first-order conditions with the constraint that the valence measure of a given candidate is invariant across elections. We also impose the constraint that the valence measures estimated in this stage satisfy the vote share equation estimated in the previous stage. The latter condition can be interpreted as a particular weighting scheme for the first-order conditions.

8.7 Extensions

Time-Varying q_I It is possible to extend our approach to settings in which q_I varies over time. Suppose that (1) $q_{I,t}$ evolves as a random walk as $q_{I,t} = q_{I,t-1} + \xi_t$; (2) ξ_t is revealed after the challenger makes her entry decision but before the candidates decide how much to spend, raise and save. This would be the case if the challenger makes an entry decision based on what she knows from the previous election and learns ξ_t only as she starts to compete for the seat. Under this timing assumption, P_e and $\mathbb{E}[M]$ are functions of $q_{I,t-1}$, $w_{I,t}$, $ten_{I,t}$ and X_t.

Consider estimating the vote share equation using the subset of the sample in which (1) the incumbent is contested in period t; (2) the incumbent is uncontested in period $t-1$. Using $\bar{\mathbf{s}}_U$ from one period before to substitute out $q_{I,t}$, the vote share equation can be expressed as follows:

$$
\begin{aligned}
vote_{I,t} = {} & \beta_I \ln d_{I,t} + \beta_C \ln d_{C,t} + \beta_{ten} \ln ten_{I,t} + \beta_{pt}(pt_t \times D_I) + \beta_{ue}(ue_t \times D_I \times D_t^P) \\
& + q_I(\mathbf{s}_{NC,t-1}) - g(P_e, \mathbb{E}[M]) + \xi_t + (q_{C,t} - g(P_e, \mathbb{E}[M])) + \varepsilon_t.
\end{aligned}
$$

The econometric error term is $\xi_t + (q_{C,t} - g(P_e, \mathbb{E}[M])) + \varepsilon_t$, where $\xi_t = (q_{I,t} - q_I(\mathbf{s}_{NC,t-1}))$. Given that the expectation of the error term conditional on $\mathbf{s}_t \equiv \{\mathbf{s}_{NC,t-1}, w_{I,t}, ten_{I,t}, X_t\}$ is 0, we can proceed as in the main text by regressing $\mathbb{E}[vote_{I,t}|\mathbf{s}_t]$ on the projections of the regressors on $\mathbf{s}_t$.[49] In our estimation, we assume time-invariant q_I due to data limitations.

Extensions to Settings without Uncontested Races We give a sketch of how our approach can be modified to settings with few uncontested elections, such as Senate races.

For this application, we assume that the researcher has access to auxiliary data such as polling data that directly identify the expected vote share, $\mathbb{E}_\varepsilon[vote_I]$ up to the error term

[49] Note that $\ln d_{I,t}$, $\ln d_{C,t}$, etc. are correlated with ξ_t, but $\mathbb{E}[\ln d_{I,t}|\mathbf{s}_t]$ are not. The latter is all we need to apply our method.

in the vote share equation, ε_t. An implication of this assumption is that we identify the exact realization of ε for each election as the difference between the realized vote and the expected vote share. Moreover, since Expression (13) implies that $K = \frac{1}{\sigma_\varepsilon}\mathbb{E}_\varepsilon(vote_I)$, the variable K is identified in every election.

We first show that, under the assumption that the continuation value V_I is increasing in own quality, the policy functions are invertible with respect to q_I and K. To see this, suppose, counterfactually, that q_I and q'_I ($q_I > q'_I$) spend and save the exact same amount conditional on (w_I, ten_I, X, K). Now consider the first-order condition (9) which equates the marginal cost of fund-raising to the marginal benefit of spending. The marginal cost of fund-raising is higher for q'_I than for q_I given our assumption of C_I. On the other hand, the marginal benefit of spending must be higher for q_I than for q'_I under the assumption that the continuation value is higher for q_I (note that K is fixed). This implies that the first-order condition cannot hold with equality at the same levels of spending and savings for both q_I and q'_I. It is easy to see that the q_I can be inverted from the policy function. As long as the policy functions are invertible, we can use actions of the incumbent, states and K from any past contested elections to replace out q_I.

In order to control for q_C, we focus on elections in which a challenger defeats an incumbent.[50] As the challenger who defeats an incumbent becomes an incumbent, we observe that candidate's actions in the next election as an incumbent. This implies that we can use the actions, states and K from future contested elections to replace out q_C. We can then identify the vote share equation and the values of q_I and q_C for a subset of the candidates. Once the vote share equation has been identified, it is straightforward to use the first-order conditions to identify the marginal cost of raising money and the marginal benefit of spending. Once these primitives are identified, the first-order conditions can be used to recover the valence measure of all candidates.

8.8 Simulating derivatives of v_C with respect to q_C

One key input to apply our methodology to the model is that challengers' entry decisions follow a cutoff strategy, governed by the threshold $\bar{q}_C(\mathbf{s})$. A key condition for this to hold is that $p(\mathbf{s}, q_C)v_C(\mathbf{s}, q_C)$ is increasing in q_C. Because we assume that $\pi(q_{C,m}, \mathbf{q}_{C,-m})$ is increasing in $q_{C,m}$, $p(\mathbf{s}, q_C)$ is increasing in q_C by assumption. Hence $p(\mathbf{s}, q_C)v_C(\mathbf{s}, q_C)$ is increasing in q_C when $v_C(\mathbf{s}, q_C)$ is not too decreasing in q_C. In this section, we take the

[50]Note that no observations of uncontested elections imply that entry probability is one, and hence our original approach to control for selection of q_C is no longer feasible.

set of estimated parameter values and simulate the derivative $\frac{\partial v_C}{\partial q_C}$ for each challenger in the data. We thereby argue that the condition likely holds locally in our data.

Recall that v_C is given as follows.

$$v_C(\mathbf{s},q_C) = \max_{d_C \geq 0, w'_C \geq 0} B \cdot \Pr(vote_I(\mathbf{s}, q_C) < 0.5) - C_C(w'_C + d_C, q_C) \\ + H_C(d_C) + \delta \Pr(vote_I(\mathbf{s}, q_C) < 0.5)\mathbb{E}_{X'|X}[V_I(\mathbf{s}')].$$

To numerically evaluate $\frac{\partial v_C}{\partial q_C}$ at a given $\{\mathbf{s}, q_C\}$ in the data, we need to evaluate v_C at state $\{\mathbf{s}, q'_C\}$ such that q'_C is located sufficiently close to q_C. To do so, we first compute the optimal values of d_I, d_C, fr_C and w'_C at q'_C, where fr_C and w'_C are fund-raising and saving of the challenger, respectively. We then evaluate each component of v_C with the new values of the actions.

Although the most formal way to compute the actions at q'_C is to solve the dynamic game and compute the policy function, this is obviously computationally burdensome. Hence, we instead approximate the candidates' policy functions regarding spending, saving and fund-raising as a flexible function of states, including q_C, and evaluate the value of spending, saving and fund-raising at q'_C. Specifically, using the set of contested elections, we regress d_I, d_C, fr_C and w'_C on a set of state variables and their interactions. We include as regressors q_C, $\exp(q_C)$, q_I and $\exp(q_I)$. We also include a constant, $pt \times D_I$, $ue \times D_I \times D_P$ and ten_I and we let coefficients of these variables depend on the President's term of office (in his first term or second term) and whether or not the election is a Midterm election (4 coefficients per variable). We use the predictions as an approximation of the policy function and evaluate the value of spending, saving and fund-raising at q'_C.

Once we have the actions at q'_C, we can evaluate each component of the value function at q'_C. For the continuation payoff, we use the polynomial approximation that we computed when we estimated the payoff components from the first-order condition. In Figure 9, we show the histogram of $v_C(\mathbf{s}, 1.001 * q_C) - v_C(\mathbf{s}, 0.999 * q_C)$, evaluated at s of each contested candidate in the data. We find that $\frac{\partial v_C}{\partial q_C} > 0$ for 89.2% of our sample. Moreover, of the ones whose $\frac{\partial v_C}{\partial q_C}$ is negative, the average value of $\frac{\partial v_C}{\partial q_C}$ is -0.00018, which is much smaller in magnitude than the average value of $\frac{\partial v_C}{\partial q_C}$ of those whose $\frac{\partial v_C}{\partial q_C}$ is positive (0.00034).

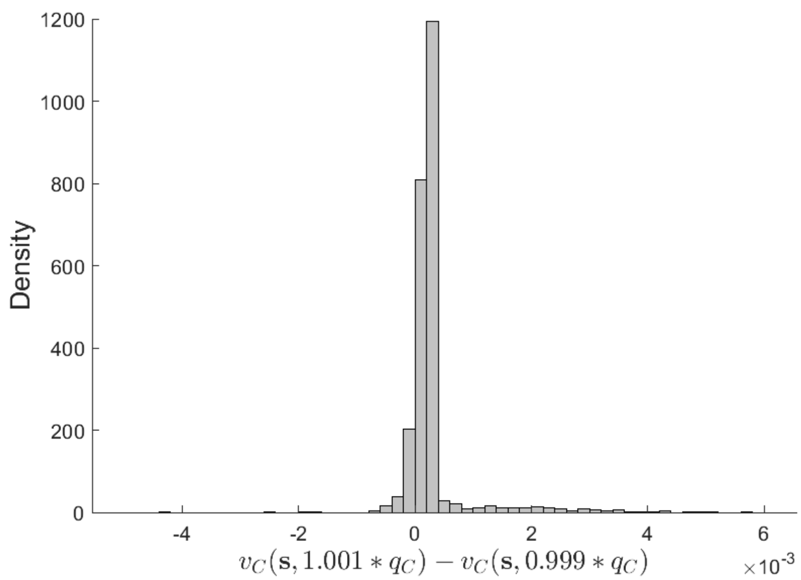

Figure 9: The derivative of v_C with respect to q_C