Detecting Large-Scale Collusion in Procurement Auctions*

Kei Kawai†  Jun Nakabayashi‡

June 21, 2018

Abstract

This paper documents evidence of widespread collusion among construction firms using a novel dataset covering most of the construction projects procured by the Japanese national government. By examining rebids that occur for auctions when all (initial) bids fail to meet the secret reserve price, we identify collusion using ideas similar to regression discontinuity. We identify about 1,000 firms whose conduct is inconsistent with competitive behavior. These bidders were awarded about 15,000 projects, or close to 40% of the total number of projects in our sample. The value of these projects totals about $19 billion.

Key words: Collusion, Procurement Auctions, Antitrust
JEL classification: D44, H57, K21, L12

1 Introduction

One of the central themes of competition policy is to deter, detect, and punish collusion. While there is almost universal agreement among economists that collusion among firms

---

*We thank Tim Armstrong, John Asker, Lanier Benkard, Philippe Gagnepain, Pat Kline, Brad Larsen, Robin Lee, Margaret Levenstein, and Larry White for helpful discussions. We especially thank Brad Larsen for suggesting ideas that led to our discussion of Section 6.1. This research has been supported by JSPS KAKENHI Grant Number 26285052.

†Department of Economics, University of California at Berkeley 530 Evans Hall #3880 Berkeley, CA, 94720-3880. Email:kei@berkeley.edu.

‡Faculty of Economics, Kindai University, Kowakae 3-4-1, Higashiosaka, 522-8502, Japan. Email: nakabayashi.1@eco.kindai.ac.jp.
is socially undesirable, firms often have private incentives to engage in collusive behavior absent regulatory sanctions. Therefore, it is crucial to ensure that the antitrust agencies have the authority and the resources to detect and punish collusion in order to promote competition among firms. To the extent that collusive activities remain undetected or unpunished, collusion may become the norm rather than the exception, with potentially large detrimental effects on the economy.

In this paper, we propose a way of detecting collusion in procurement auctions using bid pattern in reauctions. In public procurement, it is quite common for government agencies to reauction projects after an unsuccessful auction in which no bid meets the secret reserve price.\footnote{Examples in which reauctions are routinely held by the auctioneer include timber auctions by the U.S. Forest Services, procurement auctions by the State Departments of Transportation in the U.S., wireless spectrum auctions by the Federal Communication Commission, and U.S. offshore gas and oil lease auctions.} Our method of detecting collusion focuses on such projects. We then apply our method to a large dataset of procurement auctions in Japan. Our novel dataset, which covers April 2003 through December 2006, accounts for most of the construction projects procured by Japan’s national government during this period. Our data contain more than 40,000 projects, about 20% of which are put out for a bid multiple times.

Applying our method of detection to the sample of Japanese procurement auctions, we find evidence of widespread collusion that persists across regions, across types of construction projects and across time. We find a total of about 1,000 construction firms for which we reject the null hypothesis of competitive behavior. This is in contrast to the fact that the Japan Fair Trade Commission (JFTC) brought only four collusion cases (against a total of 92 firms) in connection with the procurement projects in our sample. The number of auctions won by the firms that we identify as uncompetitive totals 15,000, or close to one fifth of the total number of auctions in our sample. The total award amount of these auctions is about $19 billion. We estimate that, absent collusion by these firms, taxpayers could have saved about $1.6 billion.

The spread of collusion that we find among construction firms may have economy-wide significance. The total value of public construction projects in Japan (which includes projects procured by both local and national governments) is about $200 billion per year, or about 4% of Japan’s GDP. While our dataset accounts only for public construction projects procured at the national level, firms that we identify as uncompetitive are also active in the local procurement auctions. If we simply scale up our estimates by the size of total public construction spending, our results imply that collusive activity by construction firms affects...
about 0.85% of GDP, or 4% of total national investment. If we similarly scale up the effect on taxpayers, we find that collusion increases government spending by about $7.5 billion per year, or about 0.9% of total tax revenue.

The scale of collusion that we document in the paper also highlights the importance of rigorous enforcement of competition policy for the broader economy. Our findings lend support to the view that, absent competition policy, collusion can be widespread and affect a significant portion of an economy – as opposed to the view that collusion occurs sporadically and only under a limited set of circumstances. Our results seem to indicate that entrusting the antitrust agencies with the authority and resources to detect and punish collusion have important aggregate-level implications.

In order to illustrate our approach for detecting collusion, we begin by describing a procurement auction in New York discussed at some length in Porter and Zona (1993). In February 1983, the New York State Department of Transportation (DoT) held a procurement auction for resurfacing 0.8 miles of road. The lowest bid in the auction was $4 million, and the DoT decided not to award the contract because the bid was deemed too high relative to its own cost estimates. The project was put up for a reauction in May 1983 in which all the bidders from the initial auction participated. The lowest bid in the reauction was 20% higher than in the initial auction, submitted by the previous low bidder. Again, the contract was not awarded. The DoT held a third auction in February 1984, with the same set of bidders as in the initial auction. The lowest bid in the third auction was 10% higher than in the second time, again submitted by the same bidder. The DoT apparently thought this was suspicious: “It is notable that the same firm submitted the low bid in each of the auctions. Because of the unusual bidding patterns, the contract was not awarded through 1987.” (Porter and Zona, 1993, p. 523)

As this example illustrates, it is quite intuitive that a pattern of bidding in which the same bidder submits the lowest bid multiple times suggests collusion. If there is a bidding

---

2If we scale up our estimates by the size of total public procurement (as opposed to public construction spending), the effects will be even larger. In Japan, total public procurement accounts for approximately 13% in 2008 (OECD, 2011). Public procurement accounts for about 15% of GDP worldwide, and about 20% of GDP in OECD countries (OECD, 2011).

3Porter (2005), for example, expresses a view that is close to the former: “In any market, firms have an incentive to coordinate their decisions and increase their collective profits by restricting output and raising market prices.” For the latter view, see, e.g., Schmalensee (1987) and his description of Demsetz (1973) and (1974). In his description of the Differential Efficiency Hypothesis, Schmalensee (1987) writes, “Effective collusion is rare or nonexistent.” See also Shleifer (2005), p440.

4There is also a debate on whether or not competition policy is effective at increasing total factor productivity (see, e.g., Buccirossi et al. 2012).
ring and the project has been preallocated to one of its members, the designated winner can be expected to submit the lowest bid in the initial auction as well as in the reauctions. The detection method we propose in this paper is based on this intuition.

While the preceding argument has intuitive appeal, the key challenge in putting this argument into practice is to control for inherent cost differences across bidders. If one bidder has sufficiently low costs relative to the rest of the bidders, the lowest bidder in the initial auction may bid the lowest in subsequent reauctions even under competition. The contribution of our paper is to operationalize the preceding argument in a way that allows us to differentiate between persistence in the identity of the lowest bidder generated by inherent cost differences and persistence generated by collusive agreements. Our method is based on ideas similar to regression discontinuity: We focus on lettings with reauctions in which, in the initial auction, the second-lowest bidder loses to the lowest bidder by a very small margin. If the two lowest bids are sufficiently close, which bidder turns out to be the lowest/second-lowest bidder in the initial auction is as good as random. Then, the two bidders can be thought of as having the same costs, on average. For these lettings, the two bidders are as likely to be the lowest bidder in reauctions as the other under competition, while collusion would still predict persistence. By focusing on lettings in which the second-lowest bidder narrowly loses to the lowest bidder in the initial auction, we can separate persistence due to competition from persistence due to collusion.

Based on this idea, we present a series of simple graphical analyses as well as two formal tests of collusion. In our first graphical analysis, we study how the second-lowest bidder of the initial auction bid in the reauction relative to the lowest bidder of the initial auction. In particular, letting \( i(1) \) and \( i(2) \) denote the lowest and the second-lowest bidders of the initial auction, we plot the probability that \( i(2) \) outbids \( i(1) \) in the reauction as a function of how close the bids of \( i(1) \) and \( i(2) \) are in the initial auction. We find that the probability that \( i(2) \) outbids \( i(1) \) in the reauction is close to zero, even when we take the bid difference of \( i(1) \) and \( i(2) \) in the initial auction to zero. This is in contrast to the probability that \( i(3) \) outbids \( i(2) \) in the reauction, where \( i(3) \) is the third-lowest bidder of the initial auction. We find that the probability approaches 50% as we take the bid difference of \( i(2) \) and \( i(3) \) in the initial auction to zero.

In the second graphical analysis, we examine more closely the bidding pattern in the reauctions. In particular, we compare the distributions of two variables: the first is the distribution of \( \Delta_{12}^2 \), which is the bid difference of \( i(1) \) and \( i(2) \) in the reauction, normalized
by the reserve price. The second is the distribution of \( \Delta_{23}^2 \), which is the normalized bid difference of \( i(2) \) and \( i(3) \) in the reauction. Almost all of the mass of the distribution of \( \Delta_{12}^2 \) lies to the right of zero, reflecting the fact that a flip in the ranking of bids between \( i(1) \) and \( i(2) \) in the reauction almost never happens. The distribution of \( \Delta_{23}^2 \) is symmetric around zero.

One striking feature of the distribution of \( \Delta_{12}^2 \) is that there is what appears to be a discontinuity at exactly zero. That is, when we focus on a small band around zero, we find hundreds of lettings in which \( \Delta_{12}^2 \) falls just to the right of zero (i.e., \( \Delta_{12}^2 \in (0, t) \) for some small positive \( t \)), whereas we find very few lettings in which \( \Delta_{12}^2 \) falls just to the left of zero (i.e., \( \Delta_{12}^2 \in (-t, 0) \)). This implies that there are many reauctions in which \( i(2) \) loses to \( i(1) \) by a tiny margin, but almost no reauctions in which \( i(2) \) outbids \( i(1) \) by a tiny margin. We argue that the discontinuity in the distribution of \( \Delta_{12}^2 \) at zero suggests that the bidders are aware of how each other will bid. The distribution for \( \Delta_{23}^2 \), on the other hand, is continuous around zero with a fair amount of variance.

In order to formally test for competitive behavior, we first test for the optimality of the bidding strategy in the reauction employed by bidders who narrowly lose in the initial auction. The test captures the idea that a pattern of bidding in which \( i(2) \) is consistently outbid by \( i(1) \) in the reauction – even when \( i(2) \) is barely outbid in the initial auction – is inconsistent with optimal behavior. Our test statistic evaluates the extent to which the expected profits of the bidders who narrowly lose in the initial auction can be improved upon by playing more aggressive strategies in the reauction. Although we do not observe bidder costs, we can bound the costs of each bidder by the bid it submits in subsequent reauctions as long as the bidder bids above its costs. Given that the time between the initial auction and all subsequent auctions is very short in our setting, using subsequent bids to bound costs seems reasonable. This allows us to bound the bidders’ counterfactual profits from playing alternative strategies without having to fully characterize the equilibrium (See Haile and Tamer (2003) for a similar approach.).

Our second test is based on the smoothness of the distributions of \( \Delta_{12}^2 \) and \( \Delta_{23}^2 \). To the extent that neither \( i(1) \) nor \( i(2) \) knows how the other bidder will bid in the reauction, the difference between their bids in the reauction, \( \Delta_{12}^2 \), will have a smooth distribution. However, if \( i(1) \) and \( i(2) \) are part of a bidding ring in which \( i(1) \) is the predetermined

---

5We define \( \Delta_{12}^2 \) by subtracting the bid of \( i(1) \) from the bid of \( i(2) \) in the reauction and dividing by the reserve price. Because there is considerable variation in project size, we work with the normalized bids by dividing the actual bids by the reserve price of the auction.
winner and everybody else has agreed to bid higher than \( i(1) \) in order to yield to \( i(1) \), \( \Delta_{12} \) will be positive, and the distribution of \( \Delta_{12} \) may exhibit a point of discontinuity at zero. Our test formalizes the notion that the discontinuity is indicative of collusion. Aside from requiring different assumptions than the first test, the second test can be applied to each firm because it is less data intensive than the first.

The approach we propose in this paper may be useful to law enforcement agencies in a variety of settings. First, having a predetermined winner is quite a common feature of bidding rings. While bidding rings can be organized in a variety of ways – depending on whether or not members can engage in side-payments, explicit communication, etc. – it is common for bidding rings to pick a predetermined winner among its members beforehand and reduce competitive pressure in the actual auction. Preallocating the project to a designated bidder can also have significant cost advantages from the bidding ring’s perspective, especially in procurement auctions. Procurement auctions typically involve substantial bidding costs due to the need for estimating the costs of the project beforehand – costs that are often borne only by the designated winner in a bidding ring. That is, the non-designated bidders can avoid having to pay the project’s estimation costs. One important implication of this is that changing the designated winner between the initial auction and the reauction may be quite difficult for the bidding ring.

Second, reauctions are frequently observed in procurement settings. Examples in which the auctioneer routinely holds multiple auctions for the same object include timber auctions by the U.S. Forest Services, procurement auctions by the U.S. State DoT, and U.S. offshore gas and oil lease auctions. Because procuring agencies typically cannot commit never to try to reauction the same object when bids fail to meet the reserve price, rebidding is a very common practice in procurement auctions.

Lastly, our approach is quite simple, requiring only bidding data. Much of our analyses

---

6 Examples include the “great electrical conspiracy” (Smith, 1961; McAfee and McMillan, 1992), bidding ring of stamp dealers (Asker, 2010b), bidding ring of construction firms in Nassau and Suffolk counties (Porter and Zona, 1993), etc.

7 For example, a study by the Japan Federation of Bar Associations (JFBA) in 2001 reports that the project estimation costs are borne only by the designated winner in many bidding rings, based on facts that became clear during criminal bid-rigging cases (JFBA, 2001).

8 See McAfee and Vincent (1997) for a description of reauctions in U.S. Forest Service timber sales, Ji and Li (2008) for DoT auctions in Indiana and Porter (1995) for offshore gas and lease auctions. Ji and Li (2008) examine DoT auctions in which the auctioneer sets a secret reserve price, and there are multiple auctions for the same project when none of the bids meets the secret reserve price. In their sample, they find that about 12.5% of lettings have two rounds of bidding. In his analysis of wildcat tracts, Porter (1995) reports: “A total of 233 high bids, or 10 percent were rejected on these tracts. On the tracts with rejected bids, 47 percent were subsequently reoffered.”
does not rely on parametric assumptions on the primitives of the model.

1.1 Related Literature

This paper contributes to the empirical literature on detecting collusion in auctions.9 Existing empirical studies of collusion tend to take advantage of known episodes of cartel activity: e.g., paving in highway construction in Nassau and Suffolk counties (Porter and Zona, 1993); school milk delivery in Ohio (Porter and Zona, 1999); school milk delivery in Florida and Texas (Pesendorfer, 2000); collectible stamps in North America (Asker, 2010b); and supply of asphalt in Quebec (Clark, et al., 2018). While none of our analysis requires information on known bidding rings, it is still useful to study the bidding behavior of known cartels for validation purposes. We do this in Section 5 for the four known bidding cartels that were prosecuted by the JFTC in connection to our sample.

Another strand of literature tests for collusion in the absence of any prior knowledge of bidder conduct. Examples include seal coat contracts in three states in the U.S. Midwest (Bajari and Ye, 1999); U.S. Forest Service timber sales (Baldwin, Marshall and Richard, 1997; Athey, Levin and Seira, 2011); offshore gas and oil lease (Hendricks and Porter, 1988; Haile, Hendricks, Porter and Onuma, 2013); roadwork contracts in Italy (Conley and Decaloris, 2016); LIBOR (Abrantes-Metz et. al., 2010; Snider and Youle, 2012); public-works consulting in Okinawa in Japan (Ishii, 2009); and municipal public works in Ibaraki (Chassang and Ortner, 2017). Ishii (2009) studies 175 auctions of design consultant contracts in Naha, Okinawa and analyzes how exchange of favors can explain the winner of the auctions. Chassang and Ortner (2017) study theoretically how the introduction of minimum prices affects cartel behavior and document evidence consistent with their theoretical prediction, using data from municipalities in Ibaraki.10

The paper is also related to the literature on identification and estimation of incomplete models (See, e.g., Tamer (2011) for a survey.), and in particular, to the work of Haile and Tamer (2003). In Haile and Tamer (2003), they partially identify the distribution of bidders’ values in English auctions using the restriction that bidders do not bid above their values. Similarly, we use the idea that the costs of bidders can be bounded above by their bids in

---

9 For a brief survey, see Asker (2010a). For a more comprehensive study, see, e.g., Harrington (2008) and Marshall and Marx (2012).

10 For a more general overview of bidding rings among procurement firms in Japan, see McMillan (1991). See, also, Ohashi (2009), who discusses how the change in auction design in Mie prefecture affected collusion.
reauctions for conducting a test of collusion. The bounds on costs allow us to put bounds on the profits from playing alternative bidding strategies.

Finally, this paper is related to the literature on sequential auctions. McAfee and Vincent (1997) study the problem of a seller who can post a reserve price but cannot commit never to attempt to resell an object if it fails to sell. In a recent paper, Skreta (2013) solves for the seller’s optimal mechanism with no commitment and shows that multiple rounds of either first- or second-price auctions with optimally chosen reserve prices maximize the seller’s revenue when the bidders are ex-ante identical. The auctions in our dataset have the feature that the seller cannot commit never to resell but can commit to the same secret reserve price. Ji and Li (2008) structurally estimate a model of multi-round procurement auctions with a secret reserve price using data on procurement auctions organized by the Indiana DoT. The Indiana DoT also maintains the same secret reserve price throughout the multiple rounds as in our setting. Ji and Li (2008) recover the private cost distributions of the bidders assuming that the observed bids are competitive.

2 Institutional Background

Auction Mechanism The auction format used in our data is a first-price sealed-bid (FPSB) auction with rebidding. The auction mechanism is exactly the same as the standard FPSB auction as long as the lowest bid is below the secret reserve price, in which case, the lowest bidder becomes the winner with a price equal to the lowest bid, and the auction ends. If none of the bids is below the reserve price, however, the buyer reveals the lowest bid to all the bidders and solicits a second round of bids. The buyer reveals only the lowest bid and none of the other bids (the identity of the lowest bidder and the secret reserve price are not revealed). The second-round bidding typically takes place 30 minutes after the initial round, with the same set of bidders and the same secret reserve price. This means that when bidding in the second round, the bidders know that the secret reserve price is lower than the lowest first-round bid.

The second round proceeds in the same manner as the initial round; if the lowest bid is below the reserve price, the auction ends, and the lowest bidder wins. Otherwise, the buyer reveals the lowest second-round bid to the bidders, and the auction goes to the third round. The third round is the final round. If no bid meets the reserve price in the third round,

\footnote{See, e.g., Bidding Guidelines of Ministry of Land, Infrastructure, and Transportation, Chugoku Regional Developing Bureau.}
bilateral negotiation takes place between the buyer and the lowest third-round bidder. The same secret reserve price is used in all three rounds.

Bidding takes place online in all three rounds, and the identity of the bidders is not public at the time of bidding. The reserve price, the identity of the bidders, and all the bids in each round are made public after the auction ends.

**Collusive Behavior** In principle, bidding rings can be organized in a variety of ways, depending on whether or not members engage in side payments, whether explicit communication between the members is feasible, etc. Whatever the exact arrangement, however, a very common feature of bidding rings is that the ring picks a predetermined winner in advance and that the rest of the ring members help that predetermined winner win. The existing evidence indicates that bidding rings in the construction industry in Japan are often organized in this manner.\(^\text{12}\) The detection method that we propose in the paper exploits this feature.

Two other documented features of prosecuted bidding rings in the Japanese construction industry are worth mentioning: First, the designated winner alone typically incurs the cost of estimating the project cost.\(^\text{13}\) Estimating the project cost can be quite expensive, and the non-designated bidders typically avoid incurring this cost.\(^\text{14}\) Note that this makes it risky for a non-designated bidder to accidentally win the auction. The second feature is that the designated winner of a bidding ring would often communicate with other members in advance how they would bid in each of the three rounds (as opposed to communicating how they would bid in just the first round).\(^\text{15}\)

### 3 Data

We use a novel dataset of auctions for public construction projects obtained from the Ministry of Land, Infrastructure and Transportation (MLIT), the largest single procurement

\(^{12}\)See, for example, a report issued by the JFBA, which studies criminal bid-rigging cases (JFBA, 2001).

\(^{13}\)See, e.g., the criminal bid-rigging case regarding the construction of a sewage system in Hisai city (Tsu District Court, No. 165 (Wa), 1997); the bid-rigging case regarding the construction of a waste incineration plant in Nagoya city (Nagoya District Court, No. 1903 (Wa), 1995); etc. Based on facts that became clear in these cases, the JFBA concludes that the project estimation costs are borne only by the designated winner in many bidding rings. (JFBA (2001), p20)

\(^{14}\)Estimating the project cost involves understanding the specifications of the project, assessing the quantity and quality of materials required, negotiating prices for construction material and arranging for available subcontractors. These costs are often quite substantial.

buyer in Japan. The dataset spans April 2003 through December 2006 and covers most of the construction works auctioned by the Japanese national government during this period. After dropping scoring auctions, unit-price auctions, and those with missing or mistakenly recorded entries, we are left with 41,962 auctions with a total award amount of more than $39 billion.\textsuperscript{16}

The data include information on all bids, bidder identity, the secret reserve price, auction date, auction category (which corresponds to how restrictive bidder participation is), location of the construction site, and the type of project.\textsuperscript{17} The data also contain information on whether the auction proceeded to the second round or the third round, as well as all the bids in each round. Table 1 provides summary statistics of the data. In the table, we report the reserve price of the auction (Column (1)), the winning bid (Column (2)), the ratio of the winning bid to the reserve price (Column (3)), the lowest bid in each round as a percentage of the reserve price (Columns (4)-(6)), and the number of bidders (Column (7)).

The sample statistics are reported separately by whether the auction concluded in Round 1, Round 2, or Round 3.

In the first and second columns of the table, we find that the average reserve price of the auctions is about 99 million yen, and the average winning bid is about 94 million yen. In the third column, we find that the winning bid ranges between 93\% and 97\% of the reserve price. In the next three columns, we report the lowest bid in each round as a fraction of the reserve price. Note that for auctions that conclude in the first round, the numbers in Column (4) are equal to the number in Column (3). For auctions that conclude in the second or third round, the numbers reported in Column (4) are higher than unity by construction. Column (8) reports the sample size. We find that 20.0\% of the auctions go to the second round, and 3.0\% advance to the third round.

4 Graphical Analysis of Bidding Patterns in Reauctions

Persistence in the Second Round We begin our analysis by studying the extent to which the rank order of bids in the first round is preserved in later rounds for a given letting. Of particular interest is the extent to which the lowest bidder in the first round is also the lowest bidder in later rounds. Recall that a typical feature of bidding rings is that there is a

\textsuperscript{16}Samples with missing or mistakenly recorded entries each account for 3.0\% of the entire dataset. The scoring auction data account for 8.0\%.

\textsuperscript{17}Construction projects are divided into 21 types of construction work, such as civil engineering, architecture, bridges, paving, dredging, painting, etc.
<table>
<thead>
<tr>
<th>Concluding Round</th>
<th>(R)Reserve Yen M.</th>
<th>(W)inbid Yen M.</th>
<th>Lowest bid / Reserve</th>
<th># Bidders</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>Round 1 Round 2 Round 3</td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td>1</td>
<td>103.401</td>
<td>97.232</td>
<td>0.927</td>
<td>-</td>
<td>9.85</td>
</tr>
<tr>
<td></td>
<td>(245.72)</td>
<td>(235.62)</td>
<td>(0.085)</td>
<td>(0.085)</td>
<td>(2.60)</td>
</tr>
<tr>
<td>2</td>
<td>86.214</td>
<td>83.594</td>
<td>0.965</td>
<td>0.965</td>
<td>9.89</td>
</tr>
<tr>
<td></td>
<td>(199.24)</td>
<td>(194.51)</td>
<td>(0.033)</td>
<td>(0.075)</td>
<td>(2.42)</td>
</tr>
<tr>
<td>3</td>
<td>62.601</td>
<td>60.646</td>
<td>0.963</td>
<td>1.143</td>
<td>9.41</td>
</tr>
<tr>
<td></td>
<td>(160.66)</td>
<td>(158.34)</td>
<td>(0.034)</td>
<td>(0.113)</td>
<td>(2.26)</td>
</tr>
<tr>
<td>All</td>
<td>99.263</td>
<td>93.823</td>
<td>0.935</td>
<td>0.956</td>
<td>9.85</td>
</tr>
<tr>
<td></td>
<td>(236.46)</td>
<td>(227.29)</td>
<td>(0.079)</td>
<td>(0.103)</td>
<td>(2.56)</td>
</tr>
</tbody>
</table>

Note: The first row corresponds to the summary statistics of auctions that ended in the first round; the second row corresponds to auctions that ended in the second round; and the third row corresponds to auctions that went to the third round. The last row reports the summary statistics of all auctions. The numbers in parentheses are the standard deviations. First and second columns are in millions of yen.

| Table 1: Sample Statistics |

designated winner and that other ring members submit bids in such a way as to ensure that the designated bidder is the lowest bidder. Because the reserve price is secret in our setting, the ring members must make sure that the designated bidder is the lowest bidder in each successive round if the auction takes multiple rounds. This implies that we should observe persistence in the identity of the lowest bidder across rounds under bidder collusion.

In the top left panel of Figure 1, we report the probability that the first-round lowest bidder outbids the first-round runner-up in the reauction as a function of the differences in their first-round bids. To be precise, let $i(k)$ denote the identity of the bidder who submits the $k$-th lowest bid in Round 1, and let $b_{i(k)}^t$ denote the normalized bid of bidder $i(k)$ in round $t$. The normalized bid is the actual bid divided by the reserve price. The horizontal axis of the top left panel corresponds to $b_{i(2)}^1 - b_{i(1)}^1$ (the first-round bid difference between $i(2)$ and $i(1)$). The vertical axis corresponds to the fraction of auctions in which $b_{i(2)}^2 \geq b_{i(1)}^2$. The panel shows that, in almost all auctions, the first-round lowest bidder outbids the first-round runner-up in the reauction. The figure also shows that this fraction does not decrease as $b_{i(2)}^1 - b_{i(1)}^1$ tends to zero. Even for auctions in which $i(1)$ outbids $i(2)$ in the first round by a very small margin, $i(1)$ outbids $i(2)$ in the second round.

The fact that $i(1)$ outbids $i(2)$ in the reauction is quite natural under competition if there
Figure 1: Probability of Outbidding a Trailing Bidder. The panel plots the probability that $i(k)$ outbids $i(k+1)$ in the second round ($k \in \{1, 2, 3, 4\}$). In the top left panel, the horizontal axis corresponds to the first-round bid difference between $i(2)$ and $i(1)$. The vertical axis corresponds to the fraction of auctions in which $i(1)$ outbids $i(2)$. The other panels plot the corresponding probabilities for $i(k)$ and $i(k+1)$ with $k \in \{2, 3, 4\}$. $N$ denotes the sample size.

are inherent cost differences among the bidders. However, the fact that $i(1)$ almost always outbids $i(2)$ in the reauction even conditional on the event that $i(1)$ and $i(2)$’s first-round bids are close to being identical is hard to explain by cost heterogeneity. When the bids of $i(1)$ and $i(2)$ are almost identical, $i(1)$ and $i(2)$ should be interchangeable in terms of underlying costs.\(^{18}\)

Other panels of Figure 1 report the fraction of lettings in which $i(k)$ outbids $i(k+1)$ in the second round for $k = \{2, 3, 4\}$. Contrary to the top left panel, we find that the rank order of bids is not preserved between the trailing bidders. The panels also show that, as $b_{i(k+1)}^1 - b_{i(k)}^1$ (the first-round bid difference between $i(k+1)$ and $i(k)$) becomes smaller,

\(^{18}\)One possible difference between $i(1)$ and $i(2)$ is the information available to them. Given that only the lowest bid is announced after the first round, there is possible informational asymmetry between $i(1)$ and $i(2)$. Below, we discuss whether informational asymmetry can explain our findings.
the fraction of lettings in which $i(k+1)$ outbids $i(k)$ in the second round becomes closer to 50%.

Overall, the figure presents evidence of very high persistence in the identity of the lowest bidder. The fact that this persistence remains even when the bid differences between $i(1)$ and $i(2)$ are close to zero suggests that the persistence is unlikely to be explained away by cost differences alone. The lack of rank persistence between the trailing bidders illustrated in the other panels of Figure 1 is also consistent with this view.

**Sharp Kink in the Distribution of Second Round Bid Differences** In order to further explore the bidding pattern in the second round, the left panels of Figure 2 plot the histogram of $b_{i(2)}^2 - b_{i(1)}^2$ ($= \Delta_{12}^2$), the second-round bid difference between $i(1)$ and $i(2)$. The top left panel plots $\Delta_{12}^2$ for all auctions that reach the second round. The second and third panels in the left column plot $\Delta_{12}^2$ only for auctions in which $i(1)$ and $i(2)$ bid close to each other in the first round. In particular, the second panel conditions on $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$ and the third panel conditions on $b_{i(2)}^1 - b_{i(1)}^1 < 1\%$. The fourth panel plots $\Delta_{12}^2$ conditional on the event that the three lowest bids in the first round are all within 1%, i.e., $b_{i(3)}^1 - b_{i(1)}^1 < 1\%$.

The panels in the left column show that $\Delta_{12}^2$ falls to the right of zero almost all of the time, which confirms what we report in Figure 1: a flip in the rank order between the lowest and the second-lowest bidders almost never happens across rounds. We also confirm that the shape of $\Delta_{12}^2$ remains relatively stable across different ways of conditioning on the first-round bid differences.

In the right panels of Figure 2, we plot the histogram of $b_{i(3)}^2 - b_{i(2)}^2$ ($= \Delta_{23}^2$), the second-round bid difference between $i(2)$ and $i(3)$. Similar to the left panels, the top right panel is for all auctions that reach the second round, and the other panels correspond to conditioning on $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$, $b_{i(3)}^1 - b_{i(2)}^1 < 1\%$ and $b_{i(3)}^1 - b_{i(1)}^1 < 1\%$, respectively. In contrast to the left panels, we find that the shape of the histogram for $\Delta_{23}^2$ is symmetric around zero, implying that the rank order between $i(2)$ and $i(3)$ flips in the second round with close to 50% probability.

One striking feature of the distribution of $\Delta_{12}^2$ is that there is what appears to be a sharp kink at exactly zero. This is in stark contrast to the distribution of $\Delta_{23}^2$, which is symmetric and continuous around zero. We argue that this kink in the distribution of $\Delta_{12}^2$ is further evidence that the persistence is inconsistent with competitive behavior.

Consider, first, the distribution of $\Delta_{23}^2$ in the right panels of Figure 2. The panels show
Figure 2: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels). The first row is the histogram for the set of auctions that reach Round 2; and $i(1)$ and $i(2)$ (or $i(2)$ and $i(3)$) submit valid bids in Round 2. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small. $N$ is the sample size, and the number in the parenthesis corresponds to the fraction of auctions that lie to the left of zero. The sample sizes are different between the top left and the top right panels because in some auctions, $i(1)$ or $i(3)$ does not bid in Round 2. Similarly for the bottom left and right panels.
that even among bidders that submit almost identical first-round bids, there is a certain amount of variance in $\Delta_{23}^2$. To the extent that these bids are generated under competitive behavior, this seems to indicate that for many auctions, there is some amount of idiosyncrasy among the bidders with regard to their beliefs over the distribution of the reserve price, risk preference, etc., inducing variance in the second-round bids. In other words, idiosyncratic reasons seem to induce at least a certain amount of uncertainty in the second-round bidding for many auctions, even among bidders that submit almost identical first-round bids.

Now consider the distribution of $\Delta_{12}^2$ in the left panels of Figure 2. As long as there exists some idiosyncrasy among the bidders, $i(2)$ should outbid $i(1)$ in the second round by a narrow margin just as often as $i(1)$ outbids $i(2)$ by a narrow margin. That is, there should be a similar number of observations in which $\Delta_{12}^2 \in [-t, 0]$ and $\Delta_{12}^2 \in [0, t]$ for small values of $t$ – a feature which we clearly do not see in any of the histograms of the left panels of Figure 2. This seems inconsistent with competitive behavior.

In fact, the kink in the distribution of $\Delta_{12}^2$ at zero suggests that the bidders know exactly how the other bidders will bid in the second round. If, on the contrary, $i(1)$ and $i(2)$ were both uncertain about how each other will bid in the second round, there should be just as many cases where $i(2)$ won by a tiny margin as cases where $i(2)$ lost by a tiny margin. Hence, the discontinuity in the distribution of $\Delta_{12}^2$ suggests that the bidders have prior knowledge about each other’s bids and that $i(2)$ is deliberately losing by submitting a slightly higher bid than $i(1)$.\footnote{Our findings suggest that bidding rings determine beforehand how each ring member should bid in the second round – not just how to bid in the first round. This is natural, given that a substantial fraction of auctions go to the second round and that there are only 30 minutes between rounds.}

In Online Appendix I, we explore whether the distributions of $\Delta_{12}^2$ and $\Delta_{23}^2$ exhibit similar patterns when we condition the sample by various auction characteristics, such as region, auction category, project type, and year. We find that the distributions of $\Delta_{12}^2$ and $\Delta_{23}^2$ often look very similar to those shown in Figure 2: The distribution of $\Delta_{12}^2$ is skewed to the right and displays what appears to be a discontinuity at zero, while the distribution of $\Delta_{23}^2$ is symmetric around zero. In Online Appendix I, we also plot the second-round bid difference between $i(1)$ and $i(2)$ as well as the bid difference between $i(2)$ and $i(3)$ without normalizing the bids by the reserve price. The graphs also appear similar to Figure 2.

**Announcement of the Lowest Bid and Information Asymmetry**  
Recall from Section 2 that, at the end of each round, the lowest bid is announced, but none of the other bids are. This feature of the auction introduces asymmetry between $i(1)$ and everybody else.
We now discuss whether this asymmetry can account for the patterns illustrated in Figures 1 and 2.

Consider what each bidder learns at the end of the first round. \( i(1) \) learns that it bid the lowest bid, and that the reserve price is lower than its bid. On the other hand, the rest of the bidders learn what the lowest bid is, as well as the fact that the reserve price is lower than that. This implies that, conditional on \( i(1) \) and \( i(2) \) bidding very close to each other in the first round, the competition that \( i(1) \) believes it will face in the second round is softer than what \( i(2) \) believes it will face. To see this, consider the case in which \( i(1) \) and \( i(2) \) bid almost exactly the same amount, say \( Z \). The information revealed to \( i(1) \) at the end of the first round is that \( Z \) is the lowest bid and that it bid the lowest. The information revealed to \( i(2) \), on the other hand, is that \( Z \) is the lowest bid and that (at least) one other firm beside itself bid \( Z \). In this case, bidder \( i(1) \) expects that the most competitive rival to have bid somewhat more than its bid, on average, but the truth is that \( i(2) \) bid just as aggressively as \( i(1) \) did. This would lead bidder \( i(1) \) to underestimate the competition that it faces relative to the actual competition, which \( i(2) \) has a better sense of. This would then lead \( i(1) \) to bid less aggressively than \( i(2) \) in the second round, resulting in \( i(2) \) to outbid \( i(1) \) more than 50% of the time.

Given this discussion, the fact that the ordering between \( i(1) \) and \( i(2) \) does not change seems even less likely to occur under competition than if \( i(1) \) and \( i(2) \) had symmetric information. Conditioning on auctions in which \( i(1) \) and \( i(2) \) bid close to each other in Round 1 is similar to conditioning on the event that the actual competition that \( i(1) \) faces is tougher than what it believes to be. Hence, we would intuitively expect \( i(2) \) to bid more aggressively than \( i(1) \) and the order of \( i(1) \) and \( i(2) \) to flip more, not less, frequently than 50% under competitive behavior.

In Online Appendix II, we offer a formal treatment of this intuition by considering a simple model with two rounds and two bidder types. We first show existence of equilibria, and then show that the following is true: For every first-round bid of \( i(1) \), \( b_{i(1)}^1 \), there exists \( \varepsilon > 0 \) such that, if \( b_{i(2)}^1 - b_{i(1)}^1 < \varepsilon \), then \( i(2) \) outbids \( i(1) \) in the second round with more than 50% probability.\(^{20}\)

\(^{20}\)In the appendix, we consider a model with two rounds and two cost types. The type-symmetric equilibrium that we consider involves the low cost type to play mixed strategies in both rounds. When a bidder outbids the opponent, that bidder does not learn whether the opponent is a high cost type or a low cost type. However, if a bidder is outbid by its opponent, that bidder learns that its opponent is a low cost type. Hence, conditional on reaching the second round, a bidder who is outbid in the first round bids more aggressively in the second round, winning more than 50% of the time.
Figure 3: Plot of $b^{1}_{i(1)}$ and $b^{2}_{i(k)}$ ($k \geq 2$) for Auctions that Reach the Second Round. The left panel is a plot of $(b^{1}_{i(1)}, b^{2}_{i(k)})$ for MLIT auctions and the right panel is for municipal auctions. Note that $b^{1}_{i(1)}$ is always larger than 1 because we condition on auctions that reach the second round. In the left panel, $b^{2}_{i(k)}$ is almost always less than $b^{1}_{i(1)}$ reflecting the fact that $b^{1}_{i(1)}$ is announced to all the bidders. In the right panel, $b^{2}_{i(k)}$ is often above $b^{1}_{i(1)}$.

In order to further study whether the announcement of the lowest bid can explain the observed bidding pattern, we collected additional bidding data from three municipalities. There is significant overlap between the participants of these auctions and the participants of the baseline sample of MLIT auctions. The format of the municipal auctions is very similar to that of the MLIT auctions with one key difference: in the municipal auctions, none of the bids are announced at the end of the first round. In order to highlight this difference, Figure 3 plots $(b^{1}_{i(1)}, b^{2}_{i(k)}; k \geq 2)$, i.e., the relationship between the first-round bid of $i(1)$ and the second-round bid of $i(k)$ for MLIT auctions (left panel) and for municipal auctions (right panel) conditional on auctions that reach the second round. We find that $b^{2}_{i(k)}$ is almost always below $b^{1}_{i(1)}$ in the left panel, reflecting the fact that $i(k)$ knows $i(1)$’s first-round bid in the MLIT auctions. In the right panel, we find that there are many

---

21 About a third of the bidders in the municipal auctions also bid on the MLIT auctions.

22 There are a total of 2,848 auctions, out of which 487 auctions go to the second round (17.1%). The average number of bidders is 9.72 and the average reserve price is 19.21 million yen. The Online Appendix contains detailed sample statistics for these auctions.
Figure 4: Probability of Outbidding a Trailing Bidder for Municipal Auctions. This figure is the analogue of Figure 1 for municipal auctions. Each panel plots the probability that \( i(k) \) outbids \( i(k+1) \) in the second round as a function of \( b^1_{i(k+1)} - b^1_{i(k)} \). In the top left panel, the horizontal axis corresponds to the first-round bid difference between \( i(2) \) and \( i(1) \). The vertical axis corresponds to the fraction of auctions in which \( i(1) \) outbids \( i(2) \). The panel plots the probability that \( i(1) \) outbids \( i(2) \) in the second round. The other panels plot the corresponding probabilities for \( i(k) \) and \( i(k+1), (k \in \{2, 3, 4\}) \).

Despite the fact that the lowest bid is not revealed after the first auction in municipal auctions, the bidding pattern in the second round of the municipal auctions exhibit features that are very similar to that of MLIT auctions. Figures 4 and 5 replicate Figures 1 and 2 for municipal auctions. Figure 4 shows that \( i(1) \) outbids \( i(2) \) in the second round with very high probability and Figure 5 shows that there is a kink in the distribution of \( \Delta^2_{12} \) at zero. The fact that the bidding pattern in the MLIT and the municipal auctions exhibit similar features suggests that the announcement of the lowest bid is unlikely to be the reason for the observed bidding pattern.
Figure 5: Difference in the Second-Round Bids for Municipal Auctions. This figure is the analogue of Figure 2 for municipal auctions. The figure plots $\Delta_{12}^2$ in the left panels and $\Delta_{23}^2$ in the right panels.

**Outbidding $i(1)$ in the Second Round**  To the extent that collusive bidding implies that trailing bidders from the first round do not outbid $i(1)$ in the second round, an auction in which $i(1)$ is outbid in the second round is a sign of competition among bidders. Hence, if bidder $j$ ($\neq i(1)$) outbids $i(1)$ in the second round, we expect $j$, on average, to be a more competitive bidder than those who do not outbid $i(1)$. We explore this idea by comparing the bidding behavior of the trailing bidders who outbid $i(1)$ in the second round to those
Figure 6: Bin Plot of $b_{j,t}^{\text{before}}$ and $b_{j,t}^{\text{after}}$ Against $b_j^2 - b_{i(1)}^2$. The left panel plots $b_{j,t}^{\text{before}}$ against $b_j^2 - b_{i(1)}^2$, and the right panel plots $b_{j,t}^{\text{after}}$ against $b_j^2 - b_{i(1)}^2$. The bin size is 0.005. In both panels, the region to the left of zero corresponds to auctions in which bidder $j$ ($\neq i(1)$) outbid $i(1)$, and the region to the right of zero corresponds to auctions in which $i(1)$ outbid bidder $j$.

who do not. Specifically, for each auction $t$ and each bidder $j$, we consider the average winning bid of the five preceding and the five succeeding auctions in which bidder $j$ participates. Ordering the auctions in which bidder $j$ participates chronologically, and letting $b_{\tau}^{\text{win}}$ denote the winning bid in auction $\tau$, we consider the following two statistics:

$$b_{j,t}^{\text{before}} = \frac{1}{5} \sum_{\tau = t-5}^{t-1} b_{\tau}^{\text{win}}, \quad b_{j,t}^{\text{after}} = \frac{1}{5} \sum_{\tau = t+1}^{t+5} b_{\tau}^{\text{win}}.$$  

Note that $b_{j,t}^{\text{before}}$ and $b_{j,t}^{\text{after}}$ are defined for each auction $t$ and each bidder $j$ ($\neq i(1)$). Note also that $b_{j,t}^{\text{before}}$ and $b_{j,t}^{\text{after}}$ do not include the winning bid of the auction in question.

Figure 6 presents the results. The left panel plots the relationship between $b_{j,t}^{\text{before}}$ and $b_j^2 - b_{i(1)}^2$, i.e., the difference in the second-round bids between $i(1)$ and $j$ in auction $t$. The dots to the left of zero correspond to the average winning bid of those that trail in Round 1 and outbid $i(1)$ in Round 2. The dots to the right of zero correspond to the average winning bid of those that trail in Round 1 and lose to $i(1)$ in the second round. The panel shows that $b_{j,t}^{\text{before}}$ is lower to the left of zero than to the right of zero. The difference in $b_{j,t}^{\text{before}}$ at zero is 1.3 percentage points, and it is statistically significant at the 5% level.
The fact that there is a significant difference in \(b_{j,t}^{\text{before}}\) at zero suggests that bidders who outbid \(i(1)\) in the second round bid more competitively than those who do not in the five auctions that precede auction \(t\). This finding lends support to our claim that persistence in the identity of the lowest bidder is a sign of collusion while flips in the ranking are more likely to occur under competition. Note that when we compare \(b_{j,t}^{\text{before}}\) to the right and to the left of zero in this figure, we are comparing the bidding behavior of trailing bidders from Round 1 who outbid \(i(1)\) in Round 2 to those who do not. There is no information asymmetry between these two sets of bidders, and the results are unlikely to be explained by the announcement of the lowest bid in Round 1.

In the right panel of Figure 6, we plot \(b_{j,t}^{\text{after}}\) against \(b_j^2 - b_{i(1)}^2\). Similar to the left panel, we find that there is a significant difference in \(b_{j,t}^{\text{after}}\) of 2.7 percentage points at zero. Although this panel is also consistent with our hypothesis that persistence in the identity of the lowest bidder is symptomatic of collusion, it is not as clean as the left panel. This is because the right panel plots the winning bid of auctions that take place after the auction in question. A bidder who outbids \(i(1)\) is likely to be the winner of the auction, which introduces asymmetry between a bidder who marginally loses to \(i(1)\) and a bidder who marginally defeats \(i(1)\), from that point onward.\(^{23}\) To the extent that backlog affects future bidding behavior, the difference at zero may capture that effect.

**Persistence in the Third Round** For the subset of auctions that go to the third round, we can further examine whether a similar bidding pattern that we find for the second round continues to hold in the third round. In the top two panels of Figure 7, we plot the difference in the third-round bids of \(i(1)\) and \(i(2)\), i.e., \(\Delta_{12}^3 \equiv b_{i(2)}^3 - b_{i(1)}^3\) (left panel), and the difference in the third-round bids of \(i(2)\) and \(i(3)\), i.e., \(\Delta_{23}^3 \equiv b_{i(3)}^3 - b_{i(2)}^3\) (right panel) for all auctions that advance to the third round. In rows two to four of Figure 7, we plot the histogram conditioning on the set of auctions in which the first-round bids are relatively close. Focusing on the left panels, the second row plots \(\Delta_{12}^3\) for the set of auctions in which \(b_{i(2)}^1 - b_{i(1)}^1 < 5\%\); the third row plots \(\Delta_{12}^3\) for which \(b_{i(2)}^1 - b_{i(1)}^1 < 1\%\); and the last row plots \(\Delta_{12}^3\) for which \(b_{i(3)}^1 - b_{i(1)}^1 < 1\%\). Similarly, the second through the fourth panels in the right column plot \(\Delta_{23}^3\) for the set of auctions in which \(b_{i(3)}^1 - b_{i(2)}^1 < 5\%, b_{i(3)}^1 - b_{i(2)}^1 < 1\%\), and \(b_{i(3)}^1 - b_{i(1)}^1 < 1\%,\) respectively.

Overall, Figure 7 shows that bidding patterns in the third round are similar to that in the

\(^{23}\)Not all bidders who outbid \(i(1)\) in the second round is a winner of the auction. For example, an auction may not end in the second round, and there may be multiple bidders who outbid \(i(1)\) in the second round.
Figure 7: Difference in the Third-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Third-Round Bids of $i(2)$ and $i(3)$ (Right Panels). The first row corresponds to all auctions that reached the third round and $i(1)$ and $i(2)$ (in the case of the left panel) or $i(2)$ and $i(3)$ (in the case of the right panel) submitted valid bids in the third round. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small.
Discussion of Our Detection Method

We have discussed several ways of detecting collusion in this section. We now briefly discuss whether our idea is useful even when bidders become aware of our detection strategy.

As a general point, proposing a detection method can be thought of as putting an additional constraint on the pattern of bidding that a ring can safely engage in. Even if bidding rings respond to a new detection method, the method can still serve a useful purpose by making it potentially harder to sustain collusion, lessening the damage due to collusion, or making it easier to detect collusion with existing methods. To illustrate, one simple way for rings to avoid being detected by our method is to decrease their bids so that the auction ends in the first round. This will diminish the damage from collusion (as the bids have to be lowered, on average) and decrease the incentive to collude.

There are other ways to avoid our detection method that do not involve lowering the bids, such as changing the identity of the lowest bidder across rounds or having the lowest bidder bid substantially less than everybody else. These responses are likely to impose substantial costs on the bidding ring or make detection easier by other means. For example, if the bidding ring changed the identity of the lowest bidder from round to round, this would require at least two bidders to incur the cost of estimating the project. These additional costs would reduce the benefits of collusion, potentially making it harder to sustain it. Alternatively, if one bidder submits a bid that is substantially less than everybody else’s, this would be quite suspicious, putting the ring at risk of being detected by other methods.

5 Case Study

In this section, we analyze four collusion cases that were implicated by the JFTC during our sample period. The four cases are the bidding rings of: (A) prestressed concrete providers; (B) firms installing traffic signs; (C) builders of bridge upper structures; and (D) floodgate builders.\textsuperscript{24} In all of these cases, firms were found to have engaged in activities such as deciding on a predetermined winner for each project and communicating among the members about how each bidder will bid.\textsuperscript{25} All of the implicated firms in cases (B), (C)

\textsuperscript{24}See JFTC Recommendation #27-28 (2004) and Ruling #26-27 (2010) for case (A); JFTC Recommendation and Ruling #5-8 (2005) for case (B); JFTC Recommendation and Ruling #12 (2005) for case (C); and JFTC Cease and Desist Order #2-5 (2007) for case (D).

\textsuperscript{25}In all of these cases, the ring members took turns being the predetermined winner. The determination of who would be the predetermined winner depended on factors such as whether a given firm had an existing project that was closely related to the one being auctioned and the number of auctions a given firm had won in the recent past.
and (D) admitted wrongdoing soon after the start of the investigation, but none of the firms implicated in case (A) admitted any wrongdoing initially, and the case went to trial.\textsuperscript{26}

Before we analyze these four cases, we point out one interesting feature of the bidding ring in case (A): According to the ruling in case (A), an internal rule existed among the subset of the ring members operating in the Kansai region, which prescribed that: 1) the predetermined winner should aim to bid below the reserve price in the first round; 2) if the predetermined winner did not bid below the reserve price in the first round, it should submit a second-round bid that is less than some prespecified fraction (e.g., 0.97) of its first-round bid (e.g., $b^2_{i(1)} < 0.97 \times b^1_{i(1)}$); and 3) the rest of the ring members should submit second-round bids that are higher than the prespecified fraction of the predetermined winner’s first-round bid (e.g., $b^2_{i(k)} > 0.97 \times b^1_{i(1)}$ for $k \geq 2$). The prespecified fractions used in the ring were 0.96 for auctions with an expected value less than 100 million yen; 0.97 for auctions with an expected value between 100 million yen and 500 million yen; and 0.975 for auctions expected to be worth more than 500 million yen. One consequence of this internal rule is that we would observe the same lowest bidder in Round 1 and Round 2.

In Figure 8, we plot the winning bid against the calendar date for all auctions in which the winner is a member of one of the implicated bidding rings. We have also drawn a vertical line that corresponds to the “end date” of collusion. The “end date” is the date, according to the JFTC’s ruling, after which the ring members were deemed to have stopped colluding. The date roughly corresponds to the start date of the investigation. Note that in panels (B) and (C) of Figure 8, there exist periods after the collusion end date during which no ring member wins an auction. This reflects the fact that implicated ring members in cases (B) and (C) were banned from participating in public procurement projects for a period of up to 18 months.\textsuperscript{27}

Figure 8 shows that for cases (B), (C), and (D), there is a general drop in the winning bid of about 8.3\%, 19.5\%, and 5.3\%, respectively, after the collusion end date. However, there is almost no change in the winning bid for case (A) before and after the end date. Also, it is worth mentioning that, even for cases (B), (C), and (D), there are some auctions in which the winning bid is extremely high after the end date. In fact, about 24.4\% of auctions after the end date have a winning bid higher than 95\% for cases (B), (C) and (D). While the investigation and the ruling of the JFTC seem to have made collusion harder, it is far from

\textsuperscript{26}Out of 20 firms that were initially implicated in case (A), one firm was acquired by another firm, one was acquitted, and the rest of the firms eventually settled with the JFTC after going to trial.

\textsuperscript{27}The ring members involved in cases (A) and (D) were banned from bidding in procurement auctions for certain periods in 2010 and 2007, respectively.
clear whether the prices after the end date are truly at competitive levels. Hence, the price drops that we see in Figure 8 may be a conservative estimate of the effect of collusion. We expand on this point below.

We now examine the second-round bids of \( i(1), i(2), \) and \( i(3) \) during the period in which the firms were colluding. If the distinctive shapes of the distribution of \( \Delta_{12}^2 \) and \( \Delta_{23}^2 \) that we found in Section 4 are, indeed, evidence of collusion, we should expect to see the same pattern among the second-round bids of these colluding firms. Figure 9 plots the histogram of \( \Delta_{12}^2 \) and \( \Delta_{23}^2 \) before the collusion end date for each of the four bidding rings.
The samples used for the figure correspond to the set of auctions in which $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$ for the left column and $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$ for the right column. We see that for all four bidding rings, the histogram of $\Delta_{12}^2$ is asymmetric around zero, while the histogram of $\Delta_{23}^2$ is symmetric around zero, as before. Thus, Figure 9 suggests that the distinctive shapes of the distributions of $\Delta_{12}^2$ and $\Delta_{23}^2$ are a hallmark of collusive bidding.

We next examine the second-round bids of the ring members, but for auctions occurring after the collusion end date. To the extent that ring members stopped colluding after the end date, we should expect to see $\Delta_{12}^2$ to lie to the left of zero in a fair number of auctions. Figure 10 plots the histogram of $\Delta_{12}^2$ and $\Delta_{23}^2$ for each of the four bidding rings with $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$ and $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$. Although the sample sizes are very small, the distributions of $\Delta_{12}^2$ and $\Delta_{23}^2$ in Figure 10 are similar to those in Figure 9. That is, $\Delta_{12}^2$ is distributed to the right of zero, while $\Delta_{23}^2$ is distributed symmetrically around zero. This may seem to cast doubt on our analysis – why do the distinctive patterns in the distribution of $\Delta_{12}^2$ and $\Delta_{23}^2$ persist even after the collusion end date, when firms presumably started behaving competitively?

Our view is that the asymmetry in the distribution of $\Delta_{12}^2$ should be taken as evidence that the implicated firms were able to continue colluding at least on some auctions, even after the end date. While the bidding rings seem to have changed their behavior around the time of the end date – as the drop in the winning bid suggests in Figure 8 – this does not necessarily mean that the firms completely ceased to collude. For example, in the ruling on case (A) issued in 2010, more than five years after the start of the investigation, the judges ordered the ring members to, among other things, take various measures to prevent collusion from recurring.\(^{28}\) The judges did so because they determined that there were circumstances conducive to collusion even after the end date and that ring members needed to take steps to ensure that they did not collude in the future.\(^{29}\) Moreover, many of the firms implicated in these cases were repeat offenders. For example, one firm involved in case (A) had been found guilty in four previous collusion cases.\(^{30}\) A number of firms implicated in case (C) were also subsequently charged and found guilty of collusion in a separate case by the JFTC. It seems that being implicated by the JFTC is no guarantee that a firm will behave competitively thereafter; firms may have been able to continue colluding well beyond the end date, at least for some auctions.

\(^{28}\)JFTC Ruling #26-27 (2010). In the ruling, the firms were ordered to take preventative measures such as periodic auditing by a legal officer, etc.
With respect to case (A), there is additional evidence that the ring members continued to collude beyond the end date by following the formula for rebids that we described earlier. Recall that a subset of the prestressed concrete ring members in the Kansai region had
Figure 10: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels) After the Collusion End Date. The left panels correspond to auctions in which $b_{i(2)}^2 - b_{i(1)}^2 < 5\%$, and the right panels correspond to auctions in which $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$.

A prespecified discount (0.96 for auctions valued at less than 100 million yen; 0.97 for auctions valued between 100 million yen and 500 million yen; and 0.975 for auctions valued at more than 500 million yen) that they used when rebidding in the second round. Figure 11 plots the second-round bids of the ring members in the Kansai region as a fraction.
Figure 11: Second-Round Bids of the Ring Members of Kansai Region as a Fraction of the Lowest First-Round Bid. The top panel corresponds to auctions with a reserve price less than 100 million yen; the second panel corresponds to auctions with a reserve price between 100 million and 500 million yen; and the last panel corresponds to auctions with a reserve price above 500 million yen. The horizontal axis corresponds to the calendar date, starting from April 1, 2003. The vertical line at March 31, 2004 corresponds to the end date of the collusion for case (A). Circles correspond to $b_{i(1)}^2/b_{i(1)}^1$, and Xs correspond to $b^2_{i(k)}/b^1_{i(1)}$ for $k \geq 2$. 

29
of the lowest first-round bid. The top panel corresponds to auctions with a reserve price below 100 million yen; the middle corresponds to those with a reserve price between 100 and 500 million yen; and the last panel corresponds to those with a reserve price of more than 500 million yen. The horizontal axis in the figure corresponds to the calendar date. The vertical line in each panel corresponds to the collusion end date. Thus, auctions that took place before the end date appear to the left of this line. The circles represent $b_{i(1)}^2/b_{i(1)}^1$, and the Xs represent $b_{i(k)}^2/b_{i(1)}^1$ for $k \geq 2$. We have drawn a horizontal line at 0.96 (top panel), 0.97 (middle panel), and 0.975 (bottom panel).

While the top and the bottom panels are not very informative, note that all of $i(1)$’s second-round bids in the middle panel of Figure 11 are below 0.97 of $i(1)$’s first-round bid. Moreover, the bids of all of the others are above 0.97 of $i(1)$’s first-round bid, except for one auction. If we focus on auctions after the collusion end date, the second-round bids of $i(k)$ ($k \geq 2$) are all above 0.97. The bidding pattern in Figure 11 suggests that bidders continued to use the prespecified discount as the threshold value for submitting second-round bids. It seems quite likely that the ring members were able to maintain collusion even after the end date.

6 Formal Tests of Competitive Bidding

In this Section, we propose two tests of competitive bidding and apply them to our data. The first one tests for the optimality of the bidding strategy in the reauction employed by bidders who narrowly lose in the initial auction. The second one tests whether the shape of the distribution of $\Delta_{12}^2$ is consistent with measures of bidder idiosyncracies implied by the distribution of $\Delta_{23}^2$. The two tests differ in terms of the data and the assumptions required.

6.1 Optimality of Second-Round Bidding Strategy

Recall that there are many cases in which $i(2)$ could have outbid $i(1)$ in the second round by lowering its second-round bid by a tiny margin. For example, focusing on the left panel of the second row in Figure 2, we find that about 15.75% and 38.67% of the distribution lie within $[0, 0.01]$ and $[0, 0.02]$, respectively. On the other hand, the fraction of the distribution that lies to the left of zero is only 1.73%. This suggests that $i(2)$ can increase the probability of outbidding $i(1)$ substantially by decreasing its bid only slightly, raising the question of
whether \( i(2) \)'s second-round bid is optimal.\(^{31}\)

Based on this observation, we construct a formal test of competition. While we have found it very difficult to fully characterize the equilibrium with a continuum of bidder types under competitive bidding in our setting,\(^ {32}\) we can still test whether the observed bidding pattern is consistent with an equilibrium with competitive bidding. A necessary condition of an equilibrium is that each firm bids optimally, given the strategies played by everybody else. Thus, if we can find an alternative bidding strategy for one of the bidders that yields higher expected profits for that bidder, this implies that the observed bidding patterns are inconsistent with competitive behavior. We apply this argument to the second-round bidding strategy employed by bidders who barely lose in the first round. In particular, we show that bidders who barely lose in the first round can substantially increase their expected profits by decreasing their second-round bid by a small margin.

The key idea behind the test is that the firm’s third-round bid can provide an upper bound on its costs under private values. Using this idea, we can compute a lower bound on the bidder’s profits from playing an alternative bidding strategy in the second round without fully characterizing the equilibrium. This approach is similar in spirit to that of Haile and Tamer (2003), who obtain an upper bound on the value of bidders in an incomplete model of English auctions using the assumption that bidders do not bid above their value.

In what follows, we compare, for bidders that barely lose in the first round, the expected profits from using the current second-round strategy and the expected profits from using alternative second-round strategies. The alternative strategies that we consider are of the form \( xb_i^2 \), where \( x \) is some number less than 1 (e.g., 0.99) and \( b_i^2 \) is the bidder’s current (unnormalized) second-round bidding strategy. Just for this section, we work with the raw bids without normalizing by the reserve price. We show below that, for a range of values of \( x \), the expected profits actually increase.

First, let \( i \) be a bidder who bids slightly higher than \( i(1) \) in the first round. We will be more precise about the meaning of “slightly” later. Consider bidder \( i \)'s expected profits from using the current strategy, \( b_i^2 \), conditional on advancing to the second round. Note that the strategy (which can be a mixed strategy) depends on the information revealed to

---

\(^{31}\)Strictly speaking, \( i(2) \) does not know that it came in second at the time of rebidding (it learns only that it came close to being first). The analysis below takes this into consideration.

\(^{32}\)Because the lowest bid is revealed at the end of the first round, typically, there is no equilibrium in which everybody follows a pure monotone strategy in every round. In Online Appendix II, we characterize a mixed-strategy equilibrium with two rounds and two bidder types. However, we find it difficult to solve for the equilibrium with a continuum of bidder types.
bidder $i$ in the first round, denoted as $J$, which includes its own costs, $c_i$, its own bid, $b_1^i$, the lowest bid, $\min_j b_j^1$, and the fact that the secret reserve, $r$, is less than the lowest bid (in addition to observable auction characteristics):

$$J = (c_i, b_1^i, \min_j b_j^1, \{r < \min_j b_j^1\}).$$

The expected profits consist of two components: the expected profits from winning in the second round; and the expected profits from being the lowest bidder in the third round if the auction advances to the third round. We denote by $W^2$ the event that bidder $i$ wins in the second round and by $W^3$ the event that bidder $i$ is the lowest third-round bidder,

$$W^2 = \{b_1^i < \min \{\min_j b_j^2, r\}\}$$

$$W^3 = \{b_3^i < \min_j b_j^3, \min_j b_j^2 > r\},$$

where $b_3^i$ is bidder $i$’s current third-round bidding strategy.\(^{33}\) We now express bidder $i$’s expected profits under $b_3^i$:

$$\pi_i|J = \Pr(W^2|J)\mathbb{E}_J[b_3^i - c_i|W^2] + \Pr(W^3|J)\mathbb{E}_J[\text{profits}|W^3].$$

The profits in event $W^3$ is either $b_3^i - c_i$ if $b_3^i$ is lower than $r$, or some number less than $b_3^i - c_i$ (which depends on how the bilateral negotiation between bidder $i$ and the government plays out) if $b_3^i$ is higher than $r$. In either case, the expected profits in event $W^3$ are less than $\mathbb{E}_J[b_3^i|W^3]$. Thus, we can bound $\pi_i|J$ from above as follows:

$$\pi_i|J \leq \Pr(W^2|J)\mathbb{E}_J[b_3^i - c_i|W^2] + \Pr(W^3|J)\mathbb{E}_J[b_3^i|W^3].$$

Now consider the expected profits, $\tilde{\pi}_i|J$, from an alternative second-round bidding strategy that discounts current second-round bids by some factor $x \in (0, 1)$. As before, $\tilde{\pi}_i|J$ consists of two components, the expected profits from the second round and the expected profits from the third round:

$$\tilde{\pi}_i|J = \Pr(\tilde{W}^2|J)\mathbb{E}_J[xb_3^i - c_i|\tilde{W}^2] + \mathbb{E}_J[\text{third round profits}],$$

\(^{33}\) $b_3^i$ depends on the information available to bidder $i$ after two rounds of bidding, i.e., $(c_i, b_1^i, \min_j b_j^1, \{r < \min_j b_j^1\}, b_3^i, \min_j b_j^2, \{r < \min_j b_j^2\})$. 

32
where $\tilde{W}^2$ is the event in which bidder $i$ wins in the second round using strategy $xb^2_i$, i.e., \{ $xb^2_i < \min\{r, \min_{j \neq i} b^2_j\}$ \}. Because we are interested only in obtaining a lower bound for $\pi_i|J$ it is not necessary to specify $E_J[\text{third-round profits}]$ other than to note that it is nonnegative. Thus, we obtain a lower bound on $\tilde{\pi}_i|J$ as follows:

$$\tilde{\pi}_i|J \geq \Pr(\tilde{W}^2|J)E_{\tilde{J}}[xb^2_i - c_i|\tilde{W}^2].$$ (1)

We now compare the change in expected profits, $\Delta \pi_i|J$, from bidding $xb^2_i$ instead of $b^2_i$. Using the bound obtained above, $\Delta \pi_i|J$ can be bounded below as follows:

$$\Delta \pi_i|J \equiv \tilde{\pi}_i|J - \pi_i|J \geq \Pr(\tilde{W}^2 - W^2|J)E_{\tilde{J}}[xb^2_i - c_i|\tilde{W}^2 - W^2]$$
$$- \Pr(W^2|J)E_{J}[\{r < \min_{j \neq i} b^2_j\} - \Pr(W^3|J)E_{J}[b^3_i|W^3], (2)$$

where $\tilde{W}^2 - W^2 = \tilde{W}^2 \cap (W^2)^C$. Note that $\tilde{W}^2 - W^2$ is the event in which bidder $i$ wins in the second round with $xb^2_i$ but not with $b^2_i$. Because we consider $x \in (0, 1)$, $\tilde{W}^2$ is a superset of $W^2$, i.e., $\tilde{W}^2 \supset W^2$. The potential gain from using strategy $xb^2_i$ instead of $b^2_i$ occurs in event $\tilde{W}^2 - W^2$, and the amount of the gain is $(xb^2_i - c_i)$.\footnote{If $c_i$ is higher than $xb^2_i$, this will not be a gain, but a loss.} The first term on the right-hand side of expression (2) corresponds to the gain. The second term corresponds to the potential loss from using $xb^2_i$. In event $W^2$, using $xb^2_i$ is less profitable than $b^2_i$ because bidder $i$ is already winning with a bid of $b^2_i$. Note that a necessary condition of an equilibrium with competitive bidding is that firm $i$ has no profitable deviation, i.e., $\Delta \pi_i|J$ is nonpositive for each $J$.

In order to derive conditions that we can take to the data, consider $H$, which is a coarser partition of $J$:

$$H = (b^1_i, \min_{j \neq i} b^1_j, \{r < \min_{j \neq i} b^1_j\})$$

The difference between $J$ and $H$ is that $J$ includes $c_i$ but $H$ does not. Taking expectations of expression (2) with respect to $H$, we obtain the following expression:

$$\Delta \pi_i|H \equiv E_H[\Delta \pi_i|J]$$
$$\geq \Pr(\tilde{W}^2 - W^2|H)E_H[xb^2_i - c_i|\tilde{W}^2 - W^2]$$
$$- \Pr(W^2|H)E_H[(1 - x)b^2_i|W^2] - \Pr(W^3|H)E_H[b^3_i|W^3].$$ (3)
Figure 12: Event $\bar{W}^2 - W^2$ includes two possibilities, one in which $r$ happens to be below the lowest bid in the second round (Case 1) and the other in which $r$ happens to be above the lowest bid in the second round (Case 2). Note that $\min_j b^2_j < b^2_i$ in $\bar{W}^2 - W^2$.

Given that conditions for a competitive equilibrium require $\Delta \pi_{i|J}$ to be nonpositive for each $J$, they also require $\Delta \pi_{i|H}$ to be nonpositive for each $H$.

Taking expectations with respect to $H$ allows us to get closer to expressions that we can take to the data. All of the terms in expression (3), except for $E_H[c_i|\bar{W}^2 - W^2]$, can be evaluated directly from the data, in the sense that sample analogues can be constructed (assuming that $H$ does not include characteristics that are unobservable to the econometrician). For example, for any given value $x$, $E_H[x b^2_i|\bar{W}^2 - W^2]$ can be evaluated by taking the sample average of $x b^2_i$ for auctions in which (1) bidder $i$ bids $b^1_i$ in the first round, (2) the lowest first-round bid is $\min_j b^1_j$, and (3) a bidder does not win in the second round, but would have won if it had bid $x$ (e.g., 0.99) of the original bid. The only term that we cannot evaluate directly is $E_H[c_i|\bar{W}^2 - W^2]$ because we do not know $c_i$. However, under the private values assumption, it turns out that we can bound this term using the bidder’s third-round bid. We discuss this issue next.

Recall that $\bar{W}^2 - W^2$ corresponds to the event in which bidder $i$ wins in the second round with $x b^2_i$ but not with $b^2_i$. Event $\bar{W}^2 - W^2$ includes two possibilities, one in which $r$ happens to be below the lowest bid in the second round ($\{r < \min_j b^2_j\}$) and the other in which $r$ happens to be above ($\{r \geq \min_j b^2_j\}$). Figure 12 depicts the two situations. Note that for Case 1, the auction proceeds to the third round, and we observe $b^3_i$. Hence, we can bound $c_i$ from above by the observed third-round bid, $b^3_i$. For Case 2, however, the auction ends in the second round, and we do not observe third-round bids.

We now consider how to put bounds on $c_i$ for Case 2. Note that whether or not the auction proceeds to the third round is, to some extent, independent of the bidders’ costs. It

35Note that conditioning on $b^1_i$ and $\min_j b^1_j$ is necessary because we need to condition on $H$
depends, in part, on the random realization of \( r \). The lemma below makes this statement precise; it states that if we have two auctions with the same realizations of \( \{ b_i^1, \min_j b_j^1, b_i^2, \min_j b_j^2 \} \), but one ending in the second round and the other proceeding to the third round, bidder \( i \)'s costs must be the same, on average, in the two auctions. This lemma generalizes the observation that, if bidder \( i \) plays a pure monotone strategy, two auctions with the same realizations of \( b_i^1 \) implies the same realization of \( c_i \).  

The lemma allows us to bound bidder \( i \)'s costs for Case 2 by using the third-round bids conditional on \( H \) and \( \{ b_i^2, \min_j b_j^2 \} \).

**Lemma** Assume that bidders have private costs \( c = (c_1, \ldots, c_N) \); that \( c \) has density; and that \( c \perp r \). Then,

\[
E_H[c_i|{r \geq \min_j b_j^2}, b_i^2, \min_j b_j^2] = E_H[c_i|{b_i^2, \min_j b_j^2}] = E_H[c_i|{r < \min_j b_j^2}, b_i^2, \min_j b_j^2]. \tag{4}
\]

Moreover,

\[
E_H[c_i|{\bar{W}^2 - W^2} \cap {r \geq \min_j b_j^2}] \leq E_H[h_H(b_i^2, \min_j b_j^2)|{\bar{W}^2 - W^2} \cap {r \geq \min_j b_j^2}], \tag{5}
\]

where \( h_H(b_i^2, \min_j b_j^2) = E_H[b_i^3|\{r < \min_j b_j^2\}, b_i^2, \min_j b_j^2] \).

**Proof.** See the Appendix. ■

The first part of the lemma states that, conditional on \( \{ b_i^2, \min_j b_j^2 \} \), the expected cost of bidder \( i \) for an auction that ends in the second round given \( H \) (the first line of expression (4)) is the same as the expected cost of bidder \( i \) for an auction that goes to the third round given \( H \) (the third line of expression (4)) – under the assumption of private values and \( c \perp r \). We argue below that these two assumptions are relatively innocuous in our setting.

The second part of the lemma states that we can bound \( c_i \) in Case 2 using the mean of the observed third-round bids. Note that the left-hand side of inequality (5) is the expected bidder cost conditional on Case 2. This is bounded by the conditional expectation of \( h_H(\cdot) \), which is the expectation of \( b_i^3 \) conditional on \( \{ b_i^2, \min_j b_j^2, \{r < \min_j b_j^2\}\} \).

---

36If bidder \( i \) plays a pure monotone strategy, \( b_i^1 \) is fully revealing about \( i \)'s costs. In particular, \( E_H[c_i|{r < \min_j b_j^2}] = E_H[c_i|{r \geq \min_j b_j^2}] \) given that information set \( H \) includes \( b_i^1 \). Because we allow for mixed strategies, we need to condition on \( \{ b_i^2, \min_j b_j^2 \} \).
The reason that the Lemma is useful is that the right-hand side of expression (5) can be computed using observed data. In particular, \( h_{\mathcal{H}}(\cdot) \) can be estimated by the sample mean of the observed third-round bids conditional on \( \{b_i^2, \min_j b_j^2\} \) and \( \mathcal{H} \). In practice, our estimate of \( h_{\mathcal{H}}(\cdot) \) is a linear projection of \( b_i^3 \) on \( b_i^1, \min_j b_j^1, b_i^2, \min_j b_j^2 \) as well as auction characteristics such as year, region, and project types, based on the subset of auctions that reach the third round. We can then use the estimated function, \( h_{\mathcal{H}}(\cdot) \), to predict what the value of \( b_i^3 \) would have been for Case 2 as \( h_{\mathcal{H}}(b_i^1, \min_j b_j^2) \). The average of \( h_{\mathcal{H}}(b_i^1, \min_j b_j^2) \) among all Case 2 auctions conditional on \( \mathcal{H} \) corresponds to the right-hand side of expression (5).

Before we present our results, we briefly discuss the two assumptions of the Lemma, namely, that bidders have private values and that the costs and the reserve price are independent. We start with the private values assumption. By assuming that bidders have private values, \( c_i \) becomes constant throughout the three rounds, ensuring that \( b_i^3 \) is a valid upper bound for the costs of bidder \( i \) perceived at the time of the second round. Note that the private values assumption is sufficient but not necessary for \( b_i^3 \) to be an upper bound for \( c_i \) at Round 2. If, instead, bidders have common values, bidders may update their costs in the third round based on the observed lowest second-round bid. Our results hold as long as \( b_i^3 \) can be used as an upper bound on the costs of bidder \( i \) perceived at the time of the second round.

Next, we discuss the independence of \( c \) and \( r \). One might be inclined to argue that the independence assumption is violated based on, for example, the observation that \( c \) and \( r \) are both low for simple jobs (e.g., road paving) and that they are both high for complicated jobs (e.g., bridges). This is not necessarily a valid criticism of the independence assumption. In this example, all of the players should be aware that there are two completely different sets of distributions from which \( c \) and \( r \) are drawn, one for road paving and the other for bridges. It is not the case that there is one common set of distributions of \( c \) and \( r \) for both paving and bridges. Conditional on what is common knowledge to the players, \( c \) and \( r \) may very well be independent even in this example. As long as \( c \) and \( r \) are independent conditional on observable characteristics, the results of the Lemma remain true.

---

37Intuitively, for each auction that ends in the second round, we can find another auction that reaches the third round and has the same realization of \((b_i^1, \min_j b_j^1, b_i^2, \min_j b_j^2)\) as the former. The third-round bids of the latter can be used as a bound for \( c_i \) in the former.

38For example, suppose that bidder costs and the reserve price for paving are given by \( c_i^p = \mu^p + \epsilon_i^p \) and \( r^p = \mu^p + \epsilon^p \), and similarly for bridges, \( c_i^b = \mu^b + \epsilon_i^b \) and \( r^b = \mu^b + \epsilon^b \). Suppose, also, that \( \mu^p \) and \( \mu^b \) are constants commonly observed by the bidders, whereas \( \epsilon_i^p \) and \( \epsilon_i^b \) are privately known to the bidders. Then, \( c \) and \( r \) are independent from the perspective of the bidders as long as \( \epsilon_i^p \perp \epsilon^p \) and \( \epsilon_i^b \perp \epsilon^b \) (although they are correlated unconditionally if \( \mu^p \neq \mu^b \)).
The reason why we think that conditional independence is not a bad assumption is because of the way the reserve price is constructed. First, a project is broken down into specific procedures, each of which is then converted into a list of required input quantities. Then, for each input, the itemized price is computed by multiplying the total quantity of the input by its unit price. The reserve price is obtained by summing the itemized price for each input. Given that both the formula for converting procedures to input quantities and the unit prices are published by the auctioneer, guessing the reserve price fairly accurately is often not very difficult. However, there is rounding of itemized prices in the process of computing the reserve price, which make the reserve price random from the perspective of bidders. To the extent that the randomness in the reserve price stems from the way rounding is applied in each step, the reserve price is plausibly independent of private cost realizations at the firm level.

We are now ready to evaluate $\Delta \pi_{i|H}$, the difference in the expected profits from using $xb_{i}^{2}$ instead of $b_{i}^{2}$ in the second round. Using expressions (2) and (5) and the fact that $b_{i}^{3} > c_{i}$ for Case 1, we obtain the following bound:

$$
\Delta \pi_{i|H} \geq \Delta \pi_{i|H},
$$

where

$$
\Delta \pi_{i|H} = \Pr(\tilde{W}^{2} - W^{2}|H)E_{H}[xb_{i}^{2}\\tilde{W}^{2} - W^{2}] - \Pr(W^{3}|H)E_{H}[(1 - x)b_{i}^{2}|W^{2}]
$$

$$
- \Pr(W^{3}|H)E_{H}[b_{i}^{3}|W^{3}] - \Pr(\{\text{Case 1}\}|H)E_{H}[b_{i}^{3}|\{\text{Case 1}\}]
$$

$$
- \Pr(\{\text{Case 2}\}|H)E_{H}[h_{H}(b_{i}^{2}, \min_{j} b_{j}^{2})|\{\text{Case 2}\}];
$$

{Case 1} = (\tilde{W}^{2} - W^{2}) \cap \{r < \min_{j} b_{j}^{2}\}; \text{ and }

{Case 2} = (\tilde{W}^{2} - W^{2}) \cap \{r \geq \min_{j} b_{j}^{2}\}.

As explained earlier, we can construct sample analogues of all of the terms on the right-hand side of expression (6) and evaluate them using data. However, given that evaluating the inequality for each $H$ requires a lot of data, we work with an inequality that pools across $H$. Note that, for any coarser partition, $\mathcal{I} \subset \mathcal{H}$, we can construct an inequality analogous to expression (6) in which $H$ is replaced with $\mathcal{I}$ (except for the subscript in $h_{H}$). Hence,\[39\] We can take expectations of equation (6) with respect to $\mathcal{I}$ to obtain $\Delta \pi_{i|\mathcal{I}} = E_{\mathcal{I}}[\Delta \pi_{i|H}]$. $\Delta \pi_{i|\mathcal{I}}$ is equal to the expression in which $Pr(\cdot|H)$ is replaced by $Pr(\cdot|\mathcal{I})$ and $E_{H}$ is replaced by $E_{\mathcal{I}}$ in the expression for
Table 2: Expected Gain in Profits from Bidding $x b^2_i$. The table shows expected gain in profits from bidding $x b^2_i$ in Round 2 for bidders that lose by less than $\delta$ in Round 1. All the numbers are in Yen.

the null hypothesis that we take to the data is as follows:

$$H_0: 0 \geq \Delta \pi i|I.$$  

We report our estimates of $\Delta \pi i|I$ for $I = \{b^1_i - \min_j b^1_j < \delta \min_j b^1_j\} \cap \{r < \min_j b^1_j\}$ with three different values of $\delta$ (1%, 3%, 5%) and four different values of $x$ (99%, 98.5%, 98%, 97.5%) in Table 2. Here, $I$ also pools across auction characteristics. Note that $\{b^1_i - \min_j b^1_j < \delta \min_j b^1_j\}$ corresponds to the event that a bidder loses to the lowest bidder by less than $\delta$ in the first round. Thus, each cell in Table 2 represents the lower bound of the change in expected profits from using $xb^2$ instead of $b^2$ for all bidders who lose in the first round by less than $\delta$.

Note that all of the cells in Table 2 are positive, implying that firms would be able to increase expected profits by decreasing their second-round bids by a small margin. We reject $H_0$ with a 5% significance level for all combinations of $(x, \delta)$. In terms of magnitude, the numbers seem quite large, considering how loose our inequality is. For example, looking at $(x, \delta) = (97.5\%, 1\%)$, we see that the bidder can increase its expected profits by nearly two million yen, on average, by decreasing its second-round bids by 2.5%. Relative to the mean reserve price of 81 million yen for auctions that proceed to the second round (See Table 1), this seems substantial. Our results suggest that bidders are not bidding competitively.

\[\Delta \pi i|H^\ast.\]
\[40\text{We have implicitly assumed that both } J \text{ and } H \text{ include observable auction characteristics.}\]
Before concluding this subsection, we make a few remarks. First, it is possible to conduct the same exercise conditional on various observables, such as year, location, project type, auction category, etc., without pooling across auctions and bidders with different characteristics. In principle, we can even conduct the exercise firm by firm. However, the test is quite data intensive, because it requires many auctions that advance to the third round. The number of auctions in our dataset that reach the third round is not enough for us to apply this test firm by firm. In the next subsection, we construct an alternative test that is less data intensive, and we apply it to each firm.

Second, a test based on an inequality that pools across observables is, in some ways, stronger than a test based on an inequality that conditions on observables. If the pooled inequality is violated, then the inequality must be violated for some observables.

6.2 Firm-level Test of Collusion

In this subsection, we develop an alternative statistical test of collusive behavior that we can apply to each firm. The test is based on the idea we discussed in Section 4: the distribution of $\Delta_{12}^2$ should not be discontinuous at zero under competitive bidding.

**Test Statistic**  Recall from Section 4 that there is a reasonable amount of variance in the distribution of $\Delta_{23}^2 (= b_{i(3)}^2 - b_{i(2)}^2)$ even among auctions in which $i(2)$ and $i(3)$ submit almost identical first-round bids (See the right panels in Figure 2.). To the extent that bids are generated by competitive behavior, this means that there is a reasonable amount of bidder-specific idiosyncrasy with regard to the beliefs over the distribution of the reserve price, risk preference, etc. that induce variance in the second-round bids. This, in turn, implies that $i(1)$ cannot be outbidding $i(2)$ in the second round by a small margin all the time under competitive bidding. If $i(1)$ marginally wins some, it has to marginally lose some, depending on the realization of the bidder-specific idiosyncracy. Thus, the amount of idiosyncrasy measured by the variance of $\Delta_{23}^2$ puts a constraint on how sharply the distribution of $\Delta_{12}^2$ can change around zero. The test statistic that we propose below formalizes this idea by looking for violations of this constraint.

We begin by specifying a reduced-form model of the second-round bids of $i(2)$ and $i(3)$
when their first-round bids are very close:

\[
\begin{align*}
  b_{i(2)}^2 &= X + u_2, \\
  b_{i(3)}^2 &= X + u_3.
\end{align*}
\]

\(X\) is a common component, and \(u_2, u_3\) are bidder-specific residuals. \(X\) is a random variable whose distribution can depend on observable characteristic of the auction, information revealed in the first round, etc. Basically, \(X\) captures all of the common factors between \(i(2)\) and \(i(3)\). The residuals, \(u_2\) and \(u_3\), capture idiosyncrasies that are specific to each bidder. These residuals may reflect differences in bidders’ beliefs over the secret reserve price, heterogeneity in the bidders’ risk preferences that are privately known, etc. We assume that \(u_2\) and \(u_3\) are distributed independently and identically according to \(F_u\), and that they are independent of \(X\). Given that \(i(2)\) and \(i(3)\) are order statistics, assuming that \(u_2\) and \(u_3\) are identically distributed may seem like a strong assumption. However, as long as we condition on auctions in which the first-round bids of \(i(2)\) and \(i(3)\) are close enough, this specification seems natural, because \(i(2)\) and \(i(3)\) are almost symmetric.

Now, given that \(\Delta_{23}^2\) is just the difference between \(b_{i(3)}^2\) and \(b_{i(2)}^2\), we have

\[
\Delta_{23}^2 \equiv b_{i(3)}^2 - b_{i(2)}^2 = u_3 - u_2.
\]

Given our i.i.d. assumptions on \((u_2, u_3)\), we can recover \(F_u\) from realizations of \(\Delta_{23}^2\).

We now derive an expression that links \(F_u\) to how sharply the distribution of \(\Delta_{12}^2\) can change around zero. Let us denote by \(Y\) the second-round bid of \(i(1)\):

\[
b_{i(1)}^2 = Y.
\]

Given that \(i(1)\) has a different information set than all of the other bidders (as well as, perhaps, having different costs), we do not impose any restrictions on the distribution of \(Y\) other than independence with respect to \((u_2, u_3)\); i.e., \(Y \perp (u_2, u_3)\). In particular, \(Y\) can have arbitrary correlation with respect to \(X\).

Note that we can express \(\Delta_{12}^2\) as \(\Delta_{12}^2 = X + u_2 - Y\), given that \(\Delta_{12}^2 = b_{i(2)}^2 - b_{i(1)}^2\). The independence assumptions on \(u_2\), combined with the fact that \(F_u\) is identified, imply that

\[41\]Note that our formulation incorporates specifications such as \(b_{i(1)}^2 = Y + u_1\).
we can recover the distribution of \((Y - X), F_{Y - X}\), by deconvolution.\(^{42}\) Now, we define 
\[d(t) \equiv \Pr(\Delta_{12}^2 \in [0, t]) - \Pr(\Delta_{12}^2 \in [-t, 0]).\]

\(\Pr(\Delta_{12}^2 \in [-t, 0])\) is just the probability that \(\Delta_{12}^2\) falls within \([-t, 0]\), and \(\Pr(\Delta_{12}^2 \in [0, t])\) is the probability that \(\Delta_{12}^2\) falls within \([0, t]\). Hence, \(d(t)\) is the difference between the probability that \(\Delta_{12}^2\) falls just to the right of zero and the probability that \(\Delta_{12}^2\) falls just to the left of zero.

Under the independence assumption we have made on \((u_2, u_3)\) and \((X, Y)\), we can derive the following expression for \(d(t)\):

\[
d(t) \equiv \Pr(\Delta_{12}^2 \in [0, t]) - \Pr(\Delta_{12}^2 \in [-t, 0]) \\
= \int 1_{\{u_2 - (Y - X) \in [0, t]\}} dF_{Y - X}(Y - X) dF_u(u_2) \\
- \int 1_{\{u_2 - (Y - X) \in [-t, 0]\}} dF_{Y - X}(Y - X) dF_u(u_2) \\
= F_u(Y - X + t) - F_u(Y - X) dF_{Y - X}(Y - X) \\
- \int F_u(Y - X) - F_u(Y - X - t) dF_{Y - X}(Y - X) \\
= \int [F_u(Z + t) + F_u(Z - t) - 2F_u(Z)] dF_{Y - X}(Z),
\]

where we use the independence of \(u_2\) with respect to \(X\) and \(Y\) in the second equality. This expression relates \(d(t)\) to the smoothness of \(F_u\). To see this, note that if \(F_u\) is Lipschitz continuous with a constant \(K\), the integrand is bounded by \(tK\).\(^{43}\) Hence, the smoothness of \(F_u\) governs how large \(d(t)\) will be.

Our test statistic simply compares \(d(t)\) with the last term in the above expression. De-

---

\(^{42}\)Let \(f_{Y - X}\) and \(f_u\) denote the density of \(Y - X\) and \(u_2\) respectively. If we let \(\mathcal{F}[g]\) denote the fourier transform of \(g\), then \(\mathcal{F}[f_{\Delta_{12}^2}] = \mathcal{F}[f_{X - Y}] \mathcal{F}[f_u]\), where \(f_{\Delta_{12}^2}\) is the density of \(\Delta_{12}^2\). This implies that \(f_{X - Y} = \mathcal{F}^{-1}[\mathcal{F}[f_{\Delta_{12}^2}] / \mathcal{F}[f_u]]\). Given that \(f_{\Delta_{12}^2}\) can be directly recovered from realizations of \(\Delta_{12}^2\) and \(f_u\) can be recovered from realizations of \(\Delta_{23}^2\), we can recover \(f_{X - Y}\).

\(^{43}\)The integrand can be expressed as
\[
[F_u(Z + t) - F_u(Z)] - [F_u(Z) - F_u(Z - t)].
\]

Given that \(F_u\) is an increasing function, if \(F_u\) is Lipschitz continuous with a constant \(K\), the integrand is bounded by \(tK\).
fine $\tau(t)$ as

$$\tau(t) \equiv \left[ \int F_u(Z + t) + F_u(Z - t) - 2F_u(Z)dF_{Y - X}(Z) \right] - d(t).$$

Given that we can estimate $F_{Y - X}$, $F_u$ and $d(t)$ from realizations of $\Delta^2_{12}$ and $\Delta^2_{23}$, we can estimate $\tau(t)$. The null hypothesis that we test is $\tau(t) = 0$.

Before we apply our test, we briefly discuss how our test is related to a test of competition. To be precise, the hypothesis being tested here is a combination of 1) $u_2$ and $u_3$ are independent of each other, and they are independent of $X$ and $Y$; and 2) that $i(2)$ and $i(3)$ are identically distributed when the first-round bids of $i(2)$ and $i(3)$ are sufficiently close. Hence, a violation of the null does not immediately imply a violation of competition.

Nevertheless, we think that there is a connection between our test and a test of competition. First, it seems likely that there are many bidder idiosyncrasies that affect how bidders bid, such as how bidders update beliefs over the reserve price, risk aversion, etc., even among bidders who bid almost identical bids in the first round. Unless bidders communicate and coordinate their bids in advance, these idiosyncrasies are likely to introduce randomness in the second-round bids that are orthogonal to each other ($u_2 \perp u_3$), and to the bids of other firms ($u_2, u_3 \perp Y$). Hence, it seems likely that competition implies 1). Second, we think that 2) is relatively innocuous given that $\Delta^2_{23}$ has a distribution that is symmetric around zero when the first-round bids of $i(2)$ and $i(3)$ are close. Putting these two arguments together, we think that a violation of the null ($\tau(t) = 0$) is likely to suggest violation of 1), which, in turn, suggests that the data is not generated by competition.

We now apply this test to each firm that we observe in the data. In particular, for a given firm, we collect all auctions in which 1) the firm participates; 2) the auction proceeds to the second round; and 3) the first-round bids of $i(2)$ and $i(3)$ are sufficiently close to each other, i.e., $b^1_{i(3)} - b^1_{i(2)} < \varepsilon$. If there are more than 15 auctions that satisfy the three conditions above, we test whether or not $\Delta^2_{23}$ is symmetric around zero as a preliminary step in order to make sure that $u_2$ and $u_3$ have the same distribution. If there are more than 15 auctions and we do not reject the symmetry of $\Delta^2_{23}$, we then estimate $d(t)$, $F_u$, and $F_{X - Y}$, using realizations of $\Delta^2_{12}$ and $\Delta^2_{23}$. We use a frequency estimator for $d(t)$ and a

---

44 We test for the symmetry of $\Delta^2_{23}$ because it is implied by the assumption that $u_2$ and $u_3$ are i.i.d.. The test we use is based on the Kolmogorov-Smirnov test of identical distributions (Smirnov, 1947; Butler, 1969). In particular, we consider testing for the equality of the distribution of $\{\Delta^2_{23}: \Delta^2_{23} > 0\}$ and the distribution of $\{-\Delta^2_{23}: \Delta^2_{23} < 0\}$.

45 Note that we drop auctions if $\Delta^2_{23}$ is bigger than 30% to make sure that we exclude misrecordings, etc.
Figure 13: Estimate of $\tau(t)$ (Left Panel) and the 95 percentile of the distribution (Right Panel). We estimate $\tau(t)$ for each firm using only the subset of auctions in which it participates. The panels in the first-two rows plot the histogram for $t = 1\%$ and $t = 2\%$ with $\varepsilon = 5\%$. The panels in the bottom two rows plot the histogram for $t = 1\%$ and $t = 2\%$ with $\varepsilon = 1\%$. $N$ is the sample size.

In the top left panel of Figure 13, we plot the baseline estimates of $\tau(t)$ for each firm for $t = 1\%$ and $t = 2\%$ with $\varepsilon = 5\%$. This biases against finding collusion.

46In order to estimate $\hat{F}_u$ nonparametrically, we apply a deconvolution procedure based on Li and Vuong (1998), assuming that $F_u$ is symmetric. We compute the characteristic function of $F_u$ over the range $-10000/65$ through $10000/65$, in intervals of $1/65$. The Appendix contains a detailed explanation of our estimation procedure.
The panel shows that the estimated distribution of $\tau(t)$ lies mostly to the left of zero. Under the null hypothesis, the value of $\tau(t)$ should be zero; thus, large negative estimates of $\tau(t)$ raise concerns about possible collusive behavior. In the top right panel, we plot the estimate of the 95% quantile of $\tau(t)$ for each firm. We find that the 95% quantile of $\tau(t)$ is negative for a substantial fraction of firms. In particular, there are 449 firms for whom we reject the null of $\tau(t) = 0$ at the 95% confidence level. In the second row of Figure 13, we plot our estimate of $\tau(t)$ and the 95% quantile of $\tau(t)$ for $t = 2\%$ and $\varepsilon = 5\%$. The results are qualitatively similar. For this case, we reject the null of $\tau(t) = 0$ at the 95% confidence level for 949 firms. In the bottom two rows of Figure 13, we repeat the same exercise with $\varepsilon = 1\%$. The panels in the third row correspond to $t = 1\%$, $\varepsilon = 1\%$, and the bottom panels correspond to $t = 2\%$, $\varepsilon = 1\%$. We reject the null for 356 firms and 670 firms, respectively.

It should be clear from the construction of the test statistic that the value of $\tau(t)$ should be equal to zero for all values of $t$ under the null. Hence, we next conduct a joint hypothesis test. In particular, we pick $t = 1\%$ and $t = 2\%$ and test whether both $\tau(1\%)$ and $\tau(2\%)$ are equal to zero. Under the joint hypothesis test, we find that we can reject the null for 970 firms for $\varepsilon = 5\%$ (675 firms for $\varepsilon = 1\%$).

In order to show that our test captures the competitiveness of the bidders, we correlate the estimated $\tau$ of each firm with the average winning bid of the auctions in which the firm participates. To the extent that $\tau$ captures the competitiveness of the bidders, we should expect bidders with low values of $\tau$ to participate in auctions with high winning bids and bidders with values of $\tau$ close to zero to participate in auctions with low winning bids. Figure 14 plots the results. On the horizontal axis, we plot the estimated $\tau$ for $t = 2\%$ and $\varepsilon = 5\%$ for each firm. On the vertical axis, we plot the average winning bid of each firm, in particular, the average winning bid of auctions ending in Round 1 in which the firm participates. Because $\tau$ is computed using the set of auctions that proceed to Round 2, this ensures that there exists no mechanical relationship between $\tau$ and the average winning bid.

---

47 A total of 24,050 construction firms are observed in our analysis, among which 1,054 firms participated in at least 15 auctions that proceeded to the second round with $b_{1_i(3)} - b_{1_i(2)} < 5\%$. Among these firms, we reject the symmetry of $\Delta^{2}_{23}$ for 47 firms.

48 There are a total of 699 firms participated in at least 15 auctions that proceeded to the second round with $b_{1_i(3)} - b_{1_i(2)} < 1\%$. Among these firms, we reject the symmetry of $\Delta^{2}_{23}$ for 3 firms.

49 In practice, we estimate the joint (2-dimensional) distribution of $(\tau(1\%), \tau(2\%))$. We then simulate 500 draws of $(2 \times 1)$ random vectors according to the estimated joint distribution. We test whether there are more than 25 ($= 5\%$ of 500) draws whose elements are both positive.

50 The joint hypothesis test for $\varepsilon = 5\%$ picks out 67 firms (58 firms for $\varepsilon = 1\%$) out of 92 firms that were implicated in one of the four bid-rigging cases.
Figure 14: Plot of Average Winning Bid and $\tau$. For each firm, we compute the average winning bid of auctions ending in the first round and in which the firm participates. We plot this average against our estimate of $\tau$ ($\varepsilon = 5\%$ and $t = 2\%$). The horizontal axis corresponds to $\tau$, and the vertical axis corresponds to the average winning bid.

Figure 14 shows that firms with low values of $\tau$ participate in auctions with high winning bids, suggesting that these firms are not behaving competitively even for auctions that end in Round 1. Firms with values of $\tau$ that are closer to zero seem to participate in auctions in which the bidders bid more aggressively. In Online Appendix V, we show in regression form that the negative relationship between the two variables is robust to controlling for various bidder characteristics, sample size used to compute $\tau$, and the average winning bid of auctions that proceed to Round 2.

We now discuss the magnitude of our findings. In the joint hypothesis test, we reject the null of $\tau = 0$ for 970 firms for $\varepsilon = 5\%$. The total number of auctions awarded to these 970 firms is about 15,000, or close to 40 percent of the total number of auctions in our sample.

The total award amount of these auctions equals about $19.0$ billion. Given that the four cases we discussed in Section 5 show about an average of 8.4 percentage point drop in the winning bid after the bidding rings were implicated, we estimate that taxpayers could
have saved about $1.6 billion ($19.0 billion × 8.4%) in the absence of collusion.\footnote{Note that we are not necessarily overestimating the potential loss to taxpayers by tallying up all auctions – including those that ended in Round 1 – won by suspicious firms. This is because the 8.4% number that we use is derived from comparing the winning bids of all auctions won by the implicated firms before and after the collusion end date (It is the average drop across cases (A)-(D)). To the extent that some of the auctions prior to the collusion end date are competitive, this is reflected in the 8.4% number.}

While this is already a large number, note that the total award amount of public construction projects in Japan is about $200 billion per year, or approximately twenty times the size of our dataset. There is ample reason to believe that collusion is just as rampant in other public construction projects as in those in our dataset, given that many firms in our dataset also participate in other public construction auctions.\footnote{Also, the rules governing procurement in local procurement auctions are very similar to the ones used by the Ministry of Land, Infrastructure, and Transportation.} If we simply scale up our estimates by the size of total public construction spending, our results imply that 4% of total national investment, or 1.9% of GDP, is associated with collusive activity by construction firms. The overall impact of collusion on taxpayers is about $7.5 billion per year, or about 0.9% of total tax revenue.

\section{Conclusion}

In this paper, we document large-scale collusion among construction firms in Japan using bidding data from government procurement projects. We find evidence of collusion across regions, across types of construction projects, and across time. Our findings lend support to the view that collusion can be widespread and affect a significant portion of an economy. The detection method we propose in this paper is very simple and requires only bid data. While our test is not a definitive proof of collusion, we believe that our method can be useful for law enforcement agencies in identifying possible cases of bid rigging.

\section*{References}


Appendix

Proof of the Lemma

We first prove that the distribution of bidder $i$’s cost, $c_i$, conditional on $\{b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j, \{r < \min_j b^1_j\}\}$ is independent of whether or not the auction ends in the second round. Let $g(\cdot)$ denote the density of $c_i$ and $F_r(\cdot)$ denote the distribution function of the reserve price. Then,

$$g(c_i|b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j, \{r < \min_j b^1_j\}, \{r < \min_j b^2_j\})$$

$$= \frac{\Pr(c_i, b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j, \{r < \min_j b^1_j\}, \{r < \min_j b^2_j\})}{\Pr(\min_j b^1_j, b^2_i, \min_j b^2_j, \{r < \min_j b^1_j\}, \{r < \min_j b^2_j\})}$$

$$= \frac{\Pr(r < \min_j b^1_j, \min_j b^2_j) \times \Pr(c_i, b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j)}{\Pr(r < \min_j b^1_j, \min_j b^2_j) \times \Pr(b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j)}.$$
Given that \( r \) is independent of \( c_i \), it is also independent of \( b_i^1, \min_j b_i^1, b_i^2, \) and \( \min_j b_j^2 \). Hence, the last expression is simplified as follows:

\[
\begin{align*}
\Pr(c_i, b_i^1, \min_j b_i^1, b_i^2, \min_j b_j^2) & \times F_r(\min_j \{\min_j b_j^2\}) \\
\Pr(b_i^1, \min_j b_i^1, b_i^2, \min_j b_j^2) & \times F_r(\min_j \{\min_j b_j^2\}) \\
= & \frac{\Pr(c_i, b_i^1, \min_j b_i^1, b_i^2, \min_j b_j^2)}{\Pr(b_i^1, \min_j b_i^1, b_i^2, \min_j b_j^2)} \left( g(c_i | b_i^1, \min_j b_i^1, b_i^2, \min_j b_j^2) \right) \\
= & \frac{\Pr(c_i, b_i^1, \min_j b_i^1, b_i^2, \min_j b_j^2)}{\Pr(b_i^1, \min_j b_i^1, b_i^2, \min_j b_j^2)} \times \max\{0, F_r(\min_j b_i^1) - F_r(\min_j b_j^2)\} \\
= & g(c_i | b_i^1, \min_j b_i^1, b_i^2, \min_j b_j^2, \{\min_j b_j^2 \leq r < \min_j b_i^1\}).
\end{align*}
\]

Hence, the first part of the lemma follows.

We now show the second part. By using an argument similar to above, we have

\[
\begin{align*}
E_H[c_i | \{r \geq \min_j b_j^2\}, b_i^2, \min_j b_j^2, (\tilde{W}_2 - W_2)], \\
= & E_H[c_i | \{r < \min_j b_j^2\}, b_i^2, \min_j b_j^2].
\end{align*}
\]

Using the restriction that bidders do not bid strictly below their costs, we obtain

\[
\begin{align*}
E_H[c_i | \{r \geq \min_j b_j^2\}, b_i^2, \min_j b_j^2, (\tilde{W}_2 - W_2)], \\
\leq & E_H[b_i^3 | \{r < \min_j b_j^2\}, b_i^2, \min_j b_j^2], \\
= & h_H(b_i^3, \min_j b_j^2).
\end{align*}
\]

Integrating both sides over \( b_i^2 \) and \( \min_j b_j^2 \) for the event such that \( (\tilde{W}_2 - W_2) \cap \{r \geq \min_j b_j^2\}, H, \) we have the second part of the lemma.
For Online Publication

Online Appendix I  Analysis of Collusive Behavior by Region, Auction Category, Project Type, and Time

In this Appendix, we show that the shape of the distributions of $\Delta^2_{12}$ and $\Delta^2_{23}$ in Figure 1 is robust to conditioning on region, auction category, project type, and year. We also show that the shape of the distribution is robust to whether or not we normalize the bids by the reserve price. Note that for Figures OA.1 through OA.5, we set $\varepsilon$ equal to 5%, i.e., the figures plot auctions in which $b^1_{i(2)} - b^1_{i(1)} < 5\%$ (left panels) or $b^1_{i(3)} - b^1_{i(2)} < 5\%$ (right panels).

By Region

Figure OA.1 plots the histogram of $\Delta^2_{12}$ and $\Delta^2_{23}$ for four of the nine regions of Japan with the largest number of auctions. The regions that we show are Hokkaido, Kanto, Kansai and Chubu, in decreasing order of number of total auctions.

By Auction Category

The MLIT auctions can be divided into four categories, depending on how restrictive the participation are. The first and the second categories are the most restrictive. In these two categories, the government typically invites ten bidders from the pool of pre-qualified contractors. The difference between the two categories is that in the second category, the government chooses bidders based on contractors’ preferences over project type, project location, etc., submitted by the contractors in advance, while in the first category, the invited bidders are chosen without consideration of contractors’ preferences.53

The third and fourth categories are less restrictive. The set of potential bidders is still restricted to the pool of pre-qualified contractors, but any pre-qualified contractor can participate in the bidding. The difference between the third and fourth categories is that in the third category, the government reserves the right to exclude potential bidders from participating in the auction under certain conditions.

Figure OA.2 plots the histogram of $\Delta^2_{12}$ and $\Delta^2_{23}$ for each of the four auction categories.

53Each pre-qualified contractor submits a form to the government every two years to express its preferences over the type and location of projects it wishes to bid on.
Figure OA.1: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Region. The left panels plot $\Delta_{12}$ for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot $\Delta_{23}$ for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

**By Project Type**

In Figure OA.3, we plot the histogram of $\Delta_{12}^2$ and $\Delta_{23}^2$ for the four types of projects with the largest number of auctions. The four types of projects are civil engineering, repair
Figure OA.2: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Auction Category. The left panels plot $\Delta_{12}$ for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot $\Delta_{23}$ for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%. Category 1 corresponds to auctions with the most restrictions on participation, and category 4 corresponds to auctions with the least restrictions.

and maintenance, paving, and communication equipment, in decreasing order of number of total auctions.
Figure OA.3: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Project Type. The left panels plot $\Delta_{12}$ for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot $\Delta_{23}$ for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

By Year

In Figure OA.4, we plot the histogram of $\Delta_{12}^2$ and $\Delta_{23}^2$, by year.
Figure OA.4: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Year. The left panels plot $\Delta_{12}$ for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot $\Delta_{23}$ for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

**Raw Bids**

In Figure OA.5, we plot the raw difference in the second-round bids without normalizing by the reserve price. The left panels plot the second-round bid differences of $i(1)$ and $i(2)$.
Figure OA.5: Raw Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Raw Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels). The left panels plot the raw difference in bids for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5% of the reserve price. The right panels plot the raw difference in bids for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5% of the reserve price. The right panels plot the second-round bid differences of $i(2)$ and $i(3)$. The top panels correspond to auctions whose reserve price is between 20 and 22 million yen. The middle and bottom panels correspond to auctions with a reserve price between 60 and 66 million yen and 90 and 99 million yen, respectively. The auctions in each row roughly correspond to the 25%, 50% and 75% quantiles in terms of project size.

The length of the bandwidth we use (i.e., 2 million, 6 million, and 9 million yen, respectively) is roughly 10% of the reserve price in each row.
Online Appendix II  A Theory of Two-stage Auctions with a Secret Reserve Price

We consider an auction game with \( n \) bidders, two cost types, and two rounds. Assume that the cost type, \( c \), is either 0 or 1, with probability \( \theta \) and \( 1 - \theta \), i.e., \( \Pr(c = 0) = \theta \), \( \Pr(c = 1) = 1 - \theta \). We consider the case \( 0 < \theta < 1 - n^{-1/3} \). Let the distribution of the reserve price in the first round be uniform \([0,1]\). We assume that the seller accepts any bid in the second round with ties in the second round broken in favor of low cost types.\(^{55}\) Assume that the lowest bid in the first round is announced, but none of the other bids are.

**Proposition OA** There exists a Bayesian Nash equilibrium of the game. In particular, there exists an equilibrium in which (1) the high-cost type bids 1 in each round; (2) the low-cost type bids mixed strategies in both rounds; and (3) the first-round bidding strategy of the low-cost type, \( H(\cdot) \), is a c.d.f over support \( \left[1/2 + (1 - \theta)n^{-1}/2, 1\right] \) that is differentiable in the interior of the support, has mass at 1, and has no mass anywhere else.

**Proof.** We consider the case of \( n = 2 \) because the proof for \( n > 2 \) is almost identical. First, consider the second round of the auction following an initial round in which the two bidders submit different bids (i.e., no tie in the first round). Let \( \mu \) denote \( i(1) \)'s belief that \( i(2) \) is a low cost type. \( \mu \) is a function of \( i(1) \)'s first-round bid, \( b_{i(1)}^1 \), but we suppress this dependence for the time being. In the equilibrium that we consider, \( i(1) \) is always a low cost bidder, which implies that \( i(2) \)'s equilibrium beliefs regarding \( i(1) \)'s type is degenerate, with all the mass on low cost. Moreover, \( \mu \) will be strictly between 0 and 1. Consider the following mixed strategy, \( F_1(\tau) \), for the low-cost \( i(1) \) bidder in the second round:

\[
F_1(\tau) = \begin{cases} 
0 & \text{if } \tau < 1 - \mu, \\
\frac{\tau + \mu - 1}{\tau} & \text{if } \tau \in [1 - \mu, 1), \\
1 & \text{if } 1 \leq \tau.
\end{cases}
\]

\( F_1(\cdot) \) is a distribution with support \([1 - \mu, 1]\), with a mass of \( 1 - \mu \) at 1. Now consider the

\(^{55}\)More precisely, if a low cost type and a high cost type bid the same amount in the second round, we assume that the low cost type wins the auction. Tie-breaking rules for other cases (e.g., ties between two low cost types) turn out not to be important. Tie breaking rules for the first round will also not be important.
following mixed strategy, $F_2(\tau)$, for the low-cost $i(2)$ bidder:

$$F_2(\tau) = \begin{cases} 
0 & \text{if } \tau < 1 - \mu, \\
\frac{\tau + \mu - 1}{\tau \mu} & \text{if } \tau \in [1 - \mu, 1), \\
1 & \text{if } 1 \leq \tau.
\end{cases}$$

For the high cost type, we consider a bidding strategy in which the bidder bids 1 with probability 1. We show below that $F_1(\cdot)$, $F_2(\cdot)$, and the bidding strategy of the high cost type constitute best responses.

Consider the second-round payoff of low-cost $i(1)$ bidder when it bids $b_{i(1)}^2 = b$:

$$\pi_{i(1)}^2(b) = (1 - F_2(b))b\mu + (1 - \mu)b.$$ 

Note that $b$ is the profit margin, $\mu(1 - F_2(b))$ is the probability that the opponent is a low cost type and bids higher than $b$, and $1 - \mu$ is the probability that the opponent is a high cost type. Substituting the expression for $F_2(\cdot)$, we obtain

$$\pi_{i(1)}^2(b) = b \left[ \left(1 - \frac{b + \mu - 1}{b \mu} \right) \mu + (1 - \mu) \right],$$

$$= 1 - \mu.$$ 

Hence, $\pi_{i(1)}^2(b)$ is constant for all $b \in [1 - \mu, 1)$. Any $b \in [1 - \mu, 1]$ maximizes low-cost $i(1)$’s payoffs given $i(2)$’s strategy, $F_2(\cdot)$.

Similarly, the payoff of low-cost $i(2)$ bidder when it bids $b \in [1 - \mu, 1)$ is as follows:

$$\pi_{i(2)}^2(b) = b \left[ (1 - F_1(b)) \right],$$

$$= b \left[ \left(1 - \frac{b + \mu - 1}{b} \right) \right],$$

$$= 1 - \mu,$$

where we have used the fact that $i(2)$ believes that $i(1)$ is a low-cost type with probability 1. We find that $\pi_{i(2)}^2(b)$ is constant for $b \in [1 - \mu, 1)$. Bidding any value in $[1 - \mu, 1)$ maximizes low-cost $i(2)$’s payoffs given $i(1)$’s strategy. It is easy to see that bidding 1 with probability 1 is also a best response for the high-cost type.\(^{56}\) Hence, the strategies described above

\(^{56}\)Recall that a tie between a high cost type and a low cost type is broken in favor of the low cost type by assumption.
constitute best responses.

Now consider the second-round of the auction game following an initial round in which the bidders bid identically. We only consider the case in which the first-round tie occurs at bids equal to 1 because this is the only case that will occur with positive probability on the equilibrium path. In this case, we need not make a distinction between \(i(1)\) and \(i(2)\), because both bidders are symmetric. Letting \(\mu\) denote bidders’ belief that the opponent is a low-cost type, the following bidding strategy constitutes a best response for the low-cost types:

\[
F(\tau) = \begin{cases} 
0 & \text{if } \tau < 1 - \mu, \\
\frac{\tau + \mu - 1}{\mu} & \text{if } \tau \in [1 - \mu, 1), \\
1 & \text{if } 1 \leq \tau.
\end{cases}
\]

We now consider the first round of the auction game. Recall that we consider an equilibrium in which the low-cost types play a mixed strategy, say \(H(\cdot)\), and the high-cost types play a pure strategy (with all the mass at 1). We focus on a bidding strategy \(H(\cdot)\) such that \(H(\cdot)\) has support between \([1 - \theta/2, 1]\), is differentiable in the interior of the support, has mass at 1, and has no mass anywhere else.

The expected payoff of a low-cost bidder from bidding \(b\), \(\pi^1(b)\), is as follows:

\[
\pi^1(b) = \begin{cases} 
\theta \int_{t=b}^{1} t(1 - \mu(t))dH(t) & \text{if } b < 1 \\
(1 - \theta H(b))(1 - b) & \text{if } b \geq 1
\end{cases}
\]

The first expression corresponds to the bidder’s expected profit when \(b < 1\). The ex-
pression has three components, corresponding to three possible outcomes. The first term corresponds to the expected profit when the opponent bids lower and the auction proceeds to the second round. The second term corresponds to the case in which the opponent bids higher and the auction proceeds to the second round. The last term corresponds to the case in which the auction ends in the first round. The second expression corresponds to the bidder’s expected profit when \( b = 1 \). Note that we now make the dependence of \( \mu \) on the first-round bid explicit, as \( \mu(b) \).

The expression for \( \mu \) is given by Bayes rule as follows:

\[
\mu(t) = \begin{cases} 
\frac{\theta(1 - H(t))}{\theta(1 - H(t)) + (1 - \theta)} & \text{if } t \in [0, 1), \\
\theta H_\delta & \text{if } t = 1.
\end{cases}
\]

Substituting this expression into the expression for the expected profit, we obtain the following expression:

\[
\pi^1(b) = \theta \int_{t=0}^{t=b} \frac{1 - \theta H(t)}{1 - \theta H(t)} dH(t) + (1 - \theta)b + [1 - \theta H(b)](1 - b)b, \quad \text{if } b < 1
\]

\[
\pi^1(1) = \theta \int_{0 < t < 1} \frac{1 - \theta H(t)}{1 - \theta H(t)} dH(t) + (1 - \theta).
\]

Taking the derivative of \( \pi^1(b) (b < 1) \) we obtain the following expression:

\[
\frac{\partial \pi^1(b)}{\partial b} = \left[ \theta b \frac{1 - \theta H(b)}{1 - \theta H(b)} - \theta (1 - b)b \right] H'(b) + (1 - \theta) + [1 - \theta H(b)](1 - 2b).
\]

Setting this expression equal to zero, and solving for \( H' \), we obtain the following expression,

\[
H'(b) = \frac{[1 - \theta H(b)] [(1 - \theta) - [1 - \theta H(b)](2b - 1)]}{\theta b[(1 - \theta H(b))(1 - b) - (1 - \theta)]}.
\]  

Lemma OA-1 below guarantees that expression (OA-1) with an initial condition \( H(1 - \theta/2) = 0 \) admits a solution that is monotone increasing in range \([1 - \theta/2, 1)\) and bounded above by 1. If we define \( H(1) = 1, H(\cdot) \) will be a proper distribution function.

We wish to prove that bidding according to \( H(\cdot) \) for low cost types is a best response to other low cost types bidding \( H(\cdot) \) (and high cost types bidding 1 with probability 1). If
other low cost types bid according to $H(\cdot)$, the payoff from bidding any $b$ between $1 - \theta/2$ and 1 (i.e., $b \in [1 - \theta/2, 1]$) yields the same expected payoff to the low cost type, by construction. Below, we show that bidding exactly equal to 1 yields the same payoff as bidding just below 1 and that bidding below $1 - \theta/2$ yields lower payoffs than $\pi^1(1 - \theta/2)$.

First we show that bidding exactly equal to 1 yields the same payoff as bidding just below 1. Consider the payoff from bidding just below 1;

$$\pi^1(1 - \varepsilon) = \theta \int_{t=0}^{1-\varepsilon} t \frac{1 - \theta}{1 - \theta H(t)} dH(t) + (1 - \theta) + [1 - \theta H(1 - \varepsilon)] \varepsilon (1 - \varepsilon).$$

Taking $\varepsilon$ to zero, we find

$$\lim_{\varepsilon \to 0} \pi^1(1 - \varepsilon) = \theta \int_{0 < t < 1} t \frac{1 - \theta}{1 - \theta H(t)} dH(t) + (1 - \theta),$$

which is the same expression as $\pi^1(1)$, the payoff from bidding exactly 1. Hence, the expected payoff from bidding 1 is the same as bidding just below it.

Now consider bidding lower than $1 - \theta/2$. The payoff associated with $b$ ($b \leq 1 - \theta/2$) is as follows:

$$\pi^1(b) = \text{expected profit from first round} + \text{expected profit from second round} = b(1 - b) + b(1 - \mu).$$

Given that this expression is strictly increasing for $b \leq 1 - \theta/2$, $\pi^1(b) < \pi^1(1 - \theta/2)$ for $b < 1 - \theta/2$. 

When $n > 2$, the differential equation that defines the equilibrium strategy of a low-cost type in the first round is given by

$$H'(b) = \frac{[1 - \theta H(b) + (1 - \theta H(b))] n - 1 \theta b [(1 - \theta H(b))] n - 1 (1 - b) - (1 - \theta)^{n-1}}{(n - 1) \theta b [(1 - \theta H(b))] n - 1 (1 - b) - (1 - \theta)^{n-1}}.$$

**Corollary OA** Let $b$ be any number between 0 and 1, i.e., $b \in [0, 1]$. Assume that the lowest bid in the first round is $b$, i.e., $b^{1}_{i(1)} = b$. Then, there exists $\varepsilon > 0$ such that if $b^{1}_{i(2)} - b^{1}_{i(1)} < \varepsilon$, the probability that $i(2)$ outbids $i(1)$ in the second round is above 1/2.

**Proof.** Consider the case in which $b < 1$. Then take $\varepsilon$ so that $b + \varepsilon < 1$. When $b^{1}_{i(2)} - b^{1}_{i(1)} < \varepsilon$, both $i(1)$ and $i(2)$ are low cost types. Then the probability that $i(2)$ wins
is \((1 - \mu) + \frac{1}{2} \mu = 1 - \frac{1}{2} \mu\). Given that \(\mu \in [0, 1]\), the winning probability is higher than \(1/2\).

Now consider the case in which \(b = 1\). Let \(\varepsilon\) be any number. \(b_{i(2)}^1 - b_{i(1)}^1 < \varepsilon\) implies that \(b_{i(1)}^1 = b_{i(2)}^1 = 1\). In this case, the probability that \(i(2)\) wins is \(1/2\). □

**Lemma OA-1** Consider the following differential equation

\[
y' = \frac{[1 - \theta y][(1 - \theta)^{n-1} - [1 - \theta y]^{n-1}(2x - 1)]}{(n - 1)\theta x[(1 - \theta y)^{n-1}(1 - x) - (1 - \theta)^{n-1}]}, \tag{OA-2}
\]

with an initial condition \(y(1/2 + (1 - \theta)^{n-1}/2) = 0\). There exists a solution \(y\) that is monotone increasing in range \([1/2 + (1 - \theta)^{n-1}/2, 1]\). Moreover, \(y(1) \leq 1\).

**Proof.** Consider the denominator of expression (OA-2),

\[
D = (n - 1)\theta x[(1 - \theta y)^{n-1}(1 - x) - (1 - \theta)^{n-1}].
\]

If \(y = 0\), the square bracket term is zero if \(x = 1 - (1 - \theta)^{n-1}\). The square bracket is decreasing in both \(x\) and \(y\). Hence, for any \(x \in (1 - (1 - \theta)^{n-1}, 1]\) and \(y \in [0, 1]\), the square bracket term is strictly negative. Because \(1 - (1 - \theta)^{n-1} < 1/2 + (1 - \theta)^{n-1}/2\) when \(\theta < 1 - \frac{(1-1/3)}{2^{n-1}}\), the right hand side of expression (OA-2) is Lipschitz continuous on \((x, y) \in [1/2 + (1 - \theta)^{n-1}/2, 1] \times [0, 1]\).

Now consider the numerator of expression (OA-2),

\[
N = (1 - \theta y) \left[ (1 - \theta)^{n-1} - (1 - \theta y)^{n-1}(2x - 1) \right].
\]

Note that the term inside the square bracket is negative if

\[
y \leq \frac{1}{\theta} \left( 1 - \frac{1 - \theta}{\sqrt{2x - 1}} \right).
\]

Hence,

\[
\text{sgn}(N) = \text{sgn}(1 - \theta y)\text{sgn}\left( y - \frac{1}{\theta} \left( 1 - \frac{1 - \theta}{\sqrt{2x - 1}} \right) \right),
\]

\[
= \text{sgn}(1 - \theta y)\text{sgn}(y - f(x)),
\]

where \(f(x) = \frac{1}{\theta} \left( 1 - \frac{1 - \theta}{\sqrt{2x - 1}} \right)\).
In the region \((x, y) \in [1/2 + (1 - \theta)^{n-1}/2, 1] \times [0, 1]\), the right hand side of expression (OA-2) is positive below \(y = f(x)\) and negative above it. Note that \(f(1/2 + (1 - \theta)^{n-1}/2) = 0\), \(f(1) = 1\), and \(f'(x) > 0\) for all \(x \in (1/2 + (1 - \theta)^{n-1}/2, 1)\) and \(\theta \in (0, 1)\). Figure OA.6 illustrates the phase diagram for the case of \(\theta = 1/2\) and \(n = 2\).

Given the Lipschitz continuity of the right hand side of expression (OA-2), there exists a local solution, \(y(\cdot)\), to the initial condition problem. \(y(\cdot)\) is increasing and \(y \leq f\). To see that \(y(\cdot)\) can be extended to the interval \([1/2 + (1 - \theta)^{n-1}/2, 1]\), suppose that it can only be extended to \(\delta\), where \(\delta < 1\). Consider \(y_\delta = \lim_{x \to \delta} y(x)\). Given that \(y(\cdot)\) is monotone and bounded above by \(f\), \(y_\delta\) exists and is finite. Moreover, \(y_\delta \leq f(\delta)\). Given that the right hand side of expression (OA-2) is continuous, \(\lim_{x \to \delta} y'(x)\) also exists and is finite. Now, consider solving for expression (OA-2) with an initial condition \((x, y) = (\delta, y_\delta)\). There exists a local solution, \(y_\delta(\cdot)\), and, its value and derivative will agree with the original solution, i.e., \(y_\delta(\delta) = \lim_{x \to \delta} y(x)\) and \(\lim_{x \to \delta} y'(x) = y_\delta'(\delta)\). Hence, we get a contradiction. Finally, \(y \leq f\) implies \(y(1) \leq f(1) = 1\).

Figure OA.6: Phase Diagram of Differential Equation (OA-1) When \(\theta = 1/2\) and \(n = 2\). The solution to (OA-1) with initial value condition \(H(1 - \theta/2) = 0\), is given by \(y = y(x)\).
Table OA.1: Sample Statistics for Auctions in Municipalities

<table>
<thead>
<tr>
<th>Concluding Round</th>
<th>(R)eserve Yen M.</th>
<th>(W)inbid Yen M.</th>
<th>(W)/(R) Lowest bid / Reserve Round 1</th>
<th>Round 2</th>
<th>Round 3</th>
<th># Bidders</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.233</td>
<td>18.278</td>
<td>0.91</td>
<td>0.91</td>
<td>-</td>
<td>-</td>
<td>9.623</td>
</tr>
<tr>
<td></td>
<td>(64.05)</td>
<td>(59.31)</td>
<td>(0.117)</td>
<td>(0.117)</td>
<td>(4.62)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12.516</td>
<td>12.321</td>
<td>0.979</td>
<td>1.024</td>
<td>0.979</td>
<td>-</td>
<td>9.92</td>
</tr>
<tr>
<td></td>
<td>(21.76)</td>
<td>(21.59)</td>
<td>(0.037)</td>
<td>(0.042)</td>
<td>(0.037)</td>
<td>(4.81)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>16.533</td>
<td>16.401</td>
<td>0.985</td>
<td>1.079</td>
<td>1.043</td>
<td>0.985</td>
<td>10.59</td>
</tr>
<tr>
<td></td>
<td>(41.70)</td>
<td>(41.48)</td>
<td>(0.02)</td>
<td>(0.071)</td>
<td>(0.048)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>19.211</td>
<td>17.562</td>
<td>0.922</td>
<td>0.934</td>
<td>1.007</td>
<td>0.985</td>
<td>9.72</td>
</tr>
<tr>
<td></td>
<td>(59.83)</td>
<td>(55.59)</td>
<td>(0.11)</td>
<td>(0.121)</td>
<td>(0.053)</td>
<td>(0.02)</td>
<td></td>
</tr>
</tbody>
</table>

Note: The first row corresponds to the summary statistics of auctions that ended in the first round; the second row corresponds to auctions that ended in the second round; and the third row corresponds to auctions that went to the third round. The last row reports the summary statistics of all auctions. The numbers in parentheses are the standard deviations. First and second columns are in millions of yen.

Online Appendix III  Sample Statistics for Municipal Auctions

The sample statistics of municipal auctions that we use in Section 4 are given in Table OA.1. We report the reserve price of the auction (Column (1)), the winning bid (Column (2)), the ratio of the winning bid to the reserve price (Column (3)), the lowest bid in each round as a percentage of the reserve price (Columns (4)-(6)), and the number of bidders (Column (7)). The sample statistics are reported separately by whether the auction concluded in Round 1, Round 2, or Round 3.

Online Appendix IV  Details on the Computation of $\tau(t)$

In this section, we discuss how we compute $\tau(t)$, the test statistic we propose in Section 6 of the main text. First, we estimate the distribution of $u_2 (u_3)$ using a deconvolution procedure based on Li and Vuong (1998). Note that the characteristic functions of $\Delta_{23}^2$ and $u_2 (u_3)$ are related as follows:

$$E[e^{it\Delta_{23}^2}] = E[e^{it(u_3 - u_2)}] = E[e^{itu_3}]E[e^{-itu_2}]$$

$$= E[e^{itu_3}]E[e^{itu_2}] = E[e^{itu_2}]^2.$$
where the first equality uses the fact that $\Delta_{23}^2 = u_3 - u_2$, the second equality is by independence of $u_2$ and $u_3$ and the third equality is by assumption of symmetry of $F_u$. Using the expression above, we can express the characteristic function of $F_u$ as follows;

$$E[e^{itu_2}] = E[e^{it\Delta_{23}^2}]^{1/2}.$$ 

The pdf of $u_2$ ($u_3$), $f_u(\tau)$, can be then be expressed as follows:

$$f_u(\tau) = \frac{1}{2\pi} \int e^{-it\tau} E[e^{it\Delta_{23}^2}]^{1/2} dt,$$

where $f_u(\cdot)$ is the density of $u_2$ ($u_3$). We estimate $f_u$ by taking the empirical analogue of the above expression;

$$\hat{f}_u(\tau) = \frac{1}{2\pi} \int e^{-it\tau} \sqrt{\frac{1}{n} \sum_{j=1}^{n} \exp(it\Delta_{23}^2)} dt,$$

where $n$ is the sample size. We compute the integral numerically by evaluating the integrand at intervals of 1/65 over the range $[-10000/65, 10000/65]$ and taking the sum. We evaluate the density at 451 points, evenly spaced between $[-0.1, 0.1]$. We then obtain an estimate of the cumulative distribution function of $u_2$ ($u_3$), $\hat{F}_u(\tau)$, by taking the cumulative sum of $\hat{f}_u(\tau)$.

Second, we compute the distribution of $X - Y$.

$$E[e^{it\Delta_{12}^i}] = E[e^{it(X+u_2-Y)}] = E[e^{it(X-Y)}]E[e^{itu_2}], \quad (OA-3)$$

or

$$E[e^{it(X-Y)}] = \frac{E[e^{it\Delta_{12}^i}]}{E[e^{itu_2}]}.$$

where the first equality of (OA-3) is by definition of $\Delta_{12}^i$ and the second equality is by independence of $u_2$ and $(X - Y)$. We can use this expression to obtain an estimate of the distribution of $X - Y$:

$$\hat{f}_{X-Y}(\tau) = \frac{1}{2\pi} \int e^{-it\tau} \frac{\frac{1}{n} \sum_{j=1}^{n} \exp(it\Delta_{12}^i)}{\sqrt{\frac{1}{n} \sum_{j=1}^{n} \exp(it\Delta_{23}^2)}} dt.$$

$^57$ Note that, 99.8% of the time, $\Delta_{23}^2 = u_2 - u_3$ is less than 0.2.
Similar as before, we compute the integral numerically by evaluating the integrand at intervals of $1/65$ over the range $[-10000/65, 10000/65]$ and taking the sum. We also evaluate the density at 451 points, evenly spaced between $[-0.1, 0.1]$.

Using our estimates of $\hat{F}_u(\tau)$ and $\hat{f}_{X-Y}(\tau)$, we can evaluate the first term of $\tau(t)$, by numerical integration as

$$\frac{1}{450} \sum_{\tau = -225}^{225} \left( \hat{F}_u(\tau + t) + \hat{F}_u(\tau - t) - 2\hat{F}_u(\tau) \right) \hat{f}_{X-Y}(\tau).$$

We compute an estimate of the second term by a frequency estimate,

$$\frac{1}{n} \sum 1_{\Delta^2_{23} \in [0,t]} - \frac{1}{n} \sum 1_{\Delta^2_{23} \in [-t,0]},$$

where $1_{\{E\}}$ is an indicator function that takes the value 1 if $E$ is true and 0 otherwise.

**Online Appendix V  Relationship Between Winning Bid and $\tau$**

In this section, we show evidence that the negative relationship between the average winning bid and the estimated $\tau(t)$ illustrated in Figure 14 is robust to controlling for various bidder characteristics, sample size used to compute $\tau$, and the average winning bid of auctions that proceed to Round 2.

To do so, we regress the average winning bid of the auctions ending in Round 1 and in which the firm participates on $\tau$, for $t = \{1\%, 2\%\}$ and $\varepsilon = \{1\%, 5\%\}$. The unit of observation in the regression is a firm, and the sample consists of firms (1) that participate in at least 15 auctions that proceed to the second round; and (2) we do not reject the null that the distribution of $\Delta^2_{23}$ is symmetric around zero.

The regression that we estimate is as follows:

$$\text{WinBid}^1_i = \beta t_i + \gamma_1 \text{SampleSize}_i + \gamma_2 \text{WinBid}^2_i + \delta^\text{Region}_i + \delta^\text{ProjectType}_i + \varepsilon_i,$$

where $\text{WinBid}^1_i$ is the average winning bid of the auctions that ends in the first round and in which firm $i$ bids, $\text{WinBid}^2_i$ is the average winning bid of the auctions that proceed to the second round and in which firm $i$ bids, and $\text{SampleSize}_i$ is the sample size used to compute $\tau$. $\delta^\text{Region}_i$ denotes region dummies and $\delta^\text{ProjectType}_i$ denotes project-type dummies. There are 9 region dummies and 21 project-type dummies. A region dummy takes the value
Table OA.2: Regression Results of Winning Bid on $\tau$

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon = 5%$</th>
<th></th>
<th>$\varepsilon = 1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t = 1%$</td>
<td>$t = 2%$</td>
<td>$t = 1%$</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>-0.0323***</td>
<td>-0.0423***</td>
<td>-0.0292***</td>
</tr>
<tr>
<td></td>
<td>(0.00854)</td>
<td>(0.00588)</td>
<td>(0.00849)</td>
</tr>
<tr>
<td>SampleSize</td>
<td>6.65e-05</td>
<td>5.81e-05</td>
<td>6.98e-05</td>
</tr>
<tr>
<td></td>
<td>(3.68e-05)</td>
<td>(3.61e-05)</td>
<td>(5.29e-05)</td>
</tr>
<tr>
<td>$WinBid^2$</td>
<td>-0.00191</td>
<td>-0.0288</td>
<td>0.0198</td>
</tr>
<tr>
<td></td>
<td>(0.0375)</td>
<td>(0.0369)</td>
<td>(0.0467)</td>
</tr>
<tr>
<td>Firm Characteristics</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Observations</td>
<td>1007</td>
<td>1007</td>
<td>696</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.447</td>
<td>0.467</td>
<td>0.468</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses. The unit of observation in the regression is a firm, and the sample consists of firms (1) that participate in at least 15 auctions that proceed to the second round; and (2) we do not reject the null that the distribution of $\Delta_{23}^2$ is symmetric around zero. Firm Characteristics consist of $\delta_{i}^{Region}$ and $\delta_{i}^{ProjectType}$. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

We find that the negative relationship between the winning bid and the estimated $\tau$ is robust to controlling for firm characteristics, the number of auctions we use to compute $\tau$, and the average winning bid of auctions that end in the first round.